(1) $f = e^{x^2-y^2}\left(\sin 2xy + i \cos 2xy\right)$

(4)

$$\frac{\partial f_1}{\partial x} = e^{x^2-y^2}\, 2x \sin 2xy + e^{x^2-y^2}(\cos 2xy)\, 2y$$

$$\frac{\partial f_2}{\partial y} = e^{x^2-y^2}(-2y)\cos 2xy + e^{x^2-y^2}(-\sin 2xy)\, 2x$$

$$\frac{\partial f_1}{\partial y} = e^{x^2-y^2}(-2y)\sin 2xy + e^{x^2-y^2}\cos(2xy)\, 2x$$

$$\frac{\partial f_2}{\partial x} = e^{x^2-y^2}\, 2x \cos 2xy + e^{x^2-y^2}(-\sin 2xy)\, 2y$$

Musí platit $\frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial y}$ a $\frac{\partial f_1}{\partial y} = -\frac{\partial f_2}{\partial x}$

to platí pouze pokud $2x\sin(2xy) + 2y\cos(2xy) = 0$

a $-2y\sin(2xy) + 2x\cos(2xy) = 0$

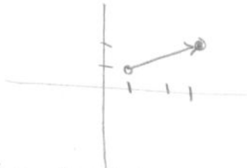
to platí pouze pro $x = y = 0$

tedy f <u>není</u> holom. na $\mathbb{C}$

(2) $\int_\gamma (1+\bar{z})^2\, dz$ $\quad\gamma$: úsečka ze $z_0 = 1+i$ do $z_1 = 3+2i$

(4)

$\gamma(t) = (1+i) + 2t + it$, $t \in [0,1]$



$$\int_\gamma (1+\bar{z})^2\, dz = \int_0^1 \left[1 + (1+2t) - i(1+t)\right]^2 \cdot (2+i)\, dt$$

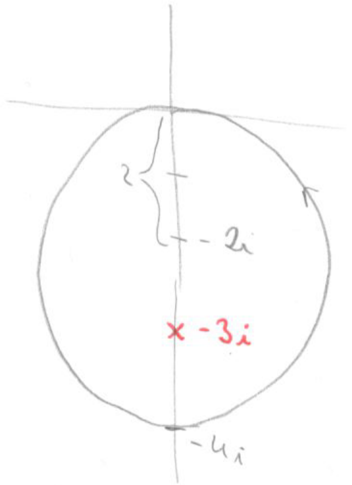
$$= \int_0^1 \left[(2-i)t + (2-i)\right]^2 (2+i)\, dt =$$

$$= \int_0^1 (10-5i)(t^2+2t+1)\, dt = (10-5i)\left[\frac{t^3}{3} + t^2 + t\right]_0^1$$

$$= \underline{(10-5i)\frac{7}{3}}$$

(3) $\displaystyle\int_\gamma \frac{\sin z}{(z+1-i)(z^2+9)}\,dz \qquad \gamma:\ |z+2i| = 2$

(4)



$$\int_\gamma \frac{\dfrac{\sin z}{(z+1-i)(z-3i)}}{(z+3i)}\,dz$$

$$= 1\cdot(2\pi i)\,\frac{\sin(-3i)}{(-3i+1-i)(-3i-3i)} =$$

$\mathrm{Ind}_\gamma(-3i)$

$$= \frac{2\pi \cancel{i}}{-6\cancel{i}}\cdot\frac{\sin(-3i)}{1-4i} = -\frac{\pi}{3}\,\frac{(1+4i)}{17}\cdot\frac{-1}{2}i\left(e^3-\frac{1}{e^3}\right)$$

$$\sin(-3i) = \frac{1}{2i}\left(e^3-e^{-3}\right) = \frac{-i}{2}\left(e^3-\frac{1}{e^3}\right)$$

$$= \frac{+\pi\left(e^3-\frac{1}{e^3}\right)}{6\cdot 17}(i-4)$$

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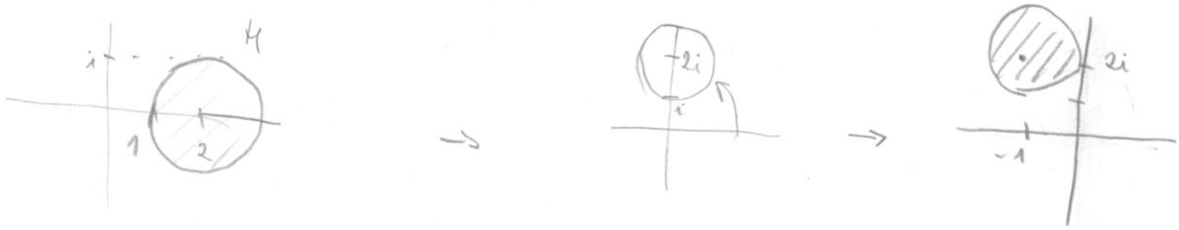
$$e^{i(-3i)} = e^3$$

$$e^{-i(-3i)} = e^{-3}$$

$$= \frac{+\pi}{102}i\left(e^3-\frac{1}{e^3}\right) - \frac{2}{51}\pi\left(e^3-\frac{1}{e^3}\right)$$

(4) $f(z) = iz-1 \qquad M:\ |z-2|\le 1$

(2)



iz : násobení č. i je otočení o $\frac{\pi}{2}$ proti směru

-1 : posun o -1 (tedy doleva)

výpočet $f(x+iy) = ix - y - 1$

např. $f(2) = 2i-1$

$f(1) = i-1$

$f(2+i) = 2i-2$