(1) $z^2 = (x+iy)^2 = \underbrace{x^2-y^2}_{f_1} + \underbrace{2xy\,i}_{f_2}$

f_1, f_2 ... polynomy mají spoj. parc. dle

(C-R) $\frac{\partial f_1}{\partial x} = 2x$ $\quad \frac{\partial f_1}{\partial y} = -2y$

$\frac{\partial f_2}{\partial x} = 2y$ $\quad \frac{\partial f_2}{\partial y} = 2x$

→ f je holomorfní

$f' = 2x + i\,2y = 2z$

(3) $f_1 = x^2 - y^2 - y$, $f(0) = 0$, $f = ?$ $\quad f \in \mathcal{H}(\mathbb{C})$

$\frac{\partial f_1}{\partial x} = 2x$ $\quad \frac{\partial f_1}{\partial y} = -2y - 1$

$= \frac{\partial f_2}{\partial y}$ $\quad \rightarrow \quad f_2 = 2xy + c(x)$

$\frac{\partial f_2}{\partial x} = 2y + c'(x) = -(-2y-1)$

$c'(x) = 1 \quad \rightarrow c(x) = x + k$

$f = x^2 - y^2 - y + i(2xy + x + k)$

$f(0) = 0 = 0 + ik \quad \Rightarrow k = 0$

$f = x^2 - y^2 - y + 2xy\,i + xi$

(2) $f_1 = x^2 - y^2 - y$ je harmonická

f_1 je polynom → má spoj parc dle 2. řádku

$\frac{\partial f_1}{\partial x} = 2x$ $\quad \frac{\partial f_1}{\partial y} = -2y - 1$

$\frac{\partial^2 f_1}{\partial x^2} = 2$ $\quad \frac{\partial^2 f_1}{\partial y^2} = -2$ ✓