

Pr. $|2-4i| = \sqrt{2^2+4^2} = \sqrt{20}$

Pr. $\overline{\left(\frac{1+i}{2-i}\right)} = \frac{(1+i)(2+i)}{2^2-i^2} = \frac{2+i+2i+i^2}{5} = \frac{1+3i}{5} = \frac{1}{5} + \frac{3}{5}i$

Pr. $-1+i = \sqrt{2} \left(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi \right)$

$| -1+i | = \sqrt{1+1} = \sqrt{2}$

$-1+i = \sqrt{2} \left(-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$

Pr.
$$\begin{aligned} (-1+i)^7 &= \left(\sqrt{2} \left(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi \right) \right)^7 \\ &= (\sqrt{2})^7 \left(\cos \frac{3 \cdot 7}{4}\pi + i \sin \frac{3}{4}\pi \cdot 7 \right) \\ &= \sqrt{2} \cdot 8 \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) \\ &= \sqrt{2} \cdot 8 \left(-\frac{1}{\sqrt{2}} + i \frac{-1}{\sqrt{2}} \right) = -8-8i \end{aligned}$$

Pr. $e^z = e^a (\cos b + i \sin b)$

$e^{i\frac{\pi}{2}+1} = e \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = ei$

$e^{-i\frac{\pi}{2}-1} = e^{-1} \left(\cos \left(-\frac{\pi}{2}\right) + i \sin \left(-\frac{\pi}{2}\right) \right) = \frac{-1}{e} i$

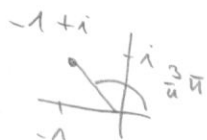
Pr. $\cos z = \frac{1}{2}(e^{iz} + e^{-iz}) \quad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz})$

$\cos \left(\frac{\pi}{2} - i \right) = \frac{1}{2} \left(e^{i\frac{\pi}{2}+1} + e^{-i\frac{\pi}{2}-1} \right) = \frac{1}{2} (ei - \frac{1}{e}i) = i \frac{1}{2}(e - \frac{1}{e})$

Pr. $\text{Arg } z = \varphi \quad \arg z = \varphi_0 \in (-\pi, \pi]$

$\text{Arg } (-1+i) = \frac{3}{4}\pi + 2k\pi, k \in \mathbb{Z}$

$\arg (-1+i) = \frac{3}{4}\pi$



Pf. $\log z = \log |z| + i \arg z$

$$\text{Log } z = \log |z| + i \text{Arg } z$$

$$\begin{aligned} \log(-1) &= \log|1| + i \arg(-1) = 0 + i\pi \\ \text{Log}(-1) &= 0 + i\pi + 2\pi i k, \quad k \in \mathbb{Z} \end{aligned}$$

Pf. $z^\alpha = e^{\alpha \log z}$

$$M_\alpha(z) = \{ e^{\alpha w} : w \in \text{Log } z \}$$

$$\begin{aligned} \sqrt{-1} &= (-1)^{1/2} = e^{\frac{1}{2} \log(-1)} = e^{\frac{1}{2} \pi i} = e^0 (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) \\ &= i \end{aligned}$$

$$\begin{aligned} M_{1/2}(-1) &= \begin{matrix} \nearrow \\ \searrow \end{matrix} e^{\frac{1}{2} 3\pi i} = e^0 (\cos \frac{3}{2} \pi + i \sin \frac{3}{2} \pi) = -i \\ &= \{ \pm i \} \end{aligned}$$