① $\lim\limits_{n\to\infty} \sqrt{3n}\cdot\dfrac{\sqrt{n^2-1}-\sqrt{n^2+n}}{\sqrt{2n+3}+\sqrt{n-1}}\cdot\dfrac{\sqrt{n^2-1}+\sqrt{n^2+n}}{\sqrt{n^2-1}+\sqrt{n^2+n}}$

$$=\lim_{n\to\infty}\sqrt{3n}\,\frac{\cancel{n^2}-1-\cancel{n^2}-n}{\sqrt{2n+3}+\sqrt{n-1}}\cdot\frac{1}{\sqrt{n^2-1}+\sqrt{n^2+n}}$$

$$=\lim_{n\to\infty}\frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}}\cdot\frac{\sqrt{3}}{\underbrace{\sqrt{2+3/n}}_{2+0\ AL}+\underbrace{\sqrt{1-1/n}}_{1-0\ AL}}\cdot\frac{\cancel{n}}{\cancel{n}}\cdot\frac{(\overset{\to 0}{-1/n}-1)}{\underbrace{\sqrt{1-1/n^2}}_{1-0\ AL}+\underbrace{\sqrt{1+1/n}}_{1+0\ AL}}$$

$$\underset{V_0\sqrt{}}{\overset{AL}{=}}\frac{\sqrt{3}}{\sqrt{2+0}+\sqrt{1-0}}\cdot\frac{0-1}{\sqrt{1-0}+\sqrt{1+0}}=\frac{-\sqrt{3}}{2(\sqrt{2}+1)}$$

② $\lim\limits_{n\to\infty}\underbrace{\sqrt[n]{3^n+2^n-\sin n+2}}_{a_n}=3$

$$\sqrt[n]{3^n}\le\sqrt[n]{3^n+2^n\underbrace{-1+2}_{1}}\le\sqrt[n]{3^n+2^n-\sin n+2}\le\sqrt[n]{3^n+2^n+1+2}\le\sqrt[n]{3^n+3^n+3^n}=\sqrt[n]{3\cdot3^n}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n}=\lim_{n\to\infty}3=3 \qquad \lim_{n\to\infty}\sqrt[n]{3\cdot3^n}=\lim_{n\to\infty}3\cdot\sqrt[n]{3}\overset{AL}{=}3\cdot1$$

ze 2 policajtů $\lim\limits_{n\to\infty}a_n=3$

(3) $\lim_{n\to\infty} \underbrace{(-1)^n \underbrace{\frac{\sqrt{n}+4^{n+1}-\log n+(-1)^n}{2^{2n}-n^3+\sqrt[4]{n}}}_{b_n}}_{a_n}$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{4^n}{4^n} \cdot \frac{\overbrace{\frac{\sqrt{n}}{4^n}}^{\to 0 \text{ škála}} + 4 - \overbrace{\frac{\log n}{4^n}}^{0 \text{ škála}} + \overbrace{\frac{(-1)^n}{4^n}}^{0 \text{ om. a miz.}}}{1 - \underbrace{\frac{n^3}{4^n}}_{0 \text{ škála}} + \underbrace{\frac{\sqrt[4]{n}}{4^n}}_{\to AL\ \frac{1}{\infty}}} \overset{AL}{=} \frac{0+4-0+0}{1-0+0} = \underline{\underline{4}}$$

z Věty o lim vybr. posl. je i $\lim b_{2n} = \lim b_{2n-1} = 4$

Dále $\lim_{n\to\infty} a_{2n} = \lim_{n\to\infty} (-1)^{2n} b_{2n} = 4$

$\lim_{n\to\infty} a_{2n-1} = \lim_{n\to\infty} (-1)^{2n-1} b_{2n-1} = -4$ $\neq$

Tedy $\lim_{n\to\infty} a_n \nexists$ (byl by to spor s jednozn. lim. a lim. vybr. posl.)

(4) (a) $\lim_{n\to\infty}(a_n+b_n)=0 \overset{?}{\Longrightarrow} \{a_n-b_n\}$ je omez.

Ne. Protipříklad $a_n = n$ $b_n = -n$

pak $a_n - b_n = 2n$

$\to$ zřejmě neom.

(b) $\{a_n-b_n\}$ je om. $\overset{?}{\Longrightarrow} \lim_{n\to\infty} a_n+b_n = 0$

Ne. Protipříklad $a_n = 3$ $b_n = 2$, $a_n+b_n=5$

$\lim_{n\to\infty} a_n+b_n=5$