(1) $\sum_{n=1}^{\infty} n^2 x^{2n}$ $\qquad \sum a_k x^k \qquad a_k = \begin{cases} 0 & k = 2n+1 \\ \left(\frac{k}{2}\right)^2 & k = 2n \end{cases}$

$\limsup_{k \to \infty} \sqrt[k]{\left(\frac{k}{2}\right)^2} = 1 \qquad \rho = 1$

$\sum$ AK na $x \in (-1,1)$, jinak diverguje. V krajích: $\sum n^2 (\pm 1)^2$ Div.

- Součet: Uvažujme $\sum_{n=1}^{\infty} n^2 y^n \qquad y = x^2$

$$f(y) = \sum_{n=1}^{\infty} n^2 y^n = y \underbrace{\sum_{n=1} n^2 y^{n-1}}_{g(y)}$$

$$G(y) = \sum_{n=1}^{\infty} n y^n = y \underbrace{\sum_{n=1} n y^{n-1}}_{h(y)}$$

$$H(y) = \sum_{n=1}^{\infty} y^n = \frac{y}{1-y} = -1 + \frac{1}{1-y}$$

$$h = H' = \frac{1}{(1-y)^2} \qquad G = \frac{y}{(1-y)^2}$$

$$g = G' = \frac{y+1}{(1-y)^3} \qquad f(y) = y \cdot \frac{y+1}{(1-y)^3}$$

$$\rightarrow \quad f(x^2) = x^2 \frac{x^2+1}{(1-x^2)^3}, \quad x \in (-1,1)$$

(2) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+2} = \frac{1}{2} \sum \frac{(-1)^n}{n+1}$

Uvažujme $f(x) = \sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1} \qquad \rho = 1 \qquad \limsup \sqrt[n]{n} = 1$

pro $x \in (-1,1)$:

$$f' = \sum_{n=1}^{\infty} x^n = \frac{x}{1-x} = -1 + \frac{1}{1-x}$$

$$f = -x + -\log|1-x| + k$$

$$f(0) = \sum 0 = 0 = -0 - \log 1 + k \to k = 0$$

$$f(x) = -x - \log|1-x| \qquad x \in (-1,1)$$

bod $x = -1$: $\quad \sum \frac{(-1)^{n+1}}{n+1}$ k (Leibniz)

$\frac{1}{n+1} \searrow 0$

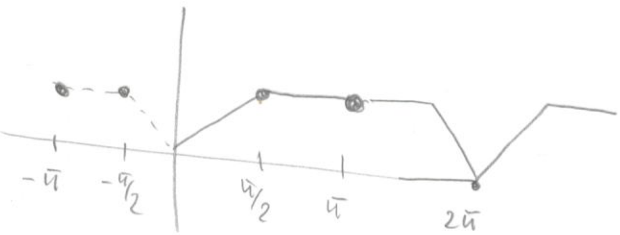
z Abelovy věty

$$f(-1) = \lim_{x \to -1+} f(x) = 1 - \log 2$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+2} = -\frac{1}{2} f(-1) = \underline{\underline{-\frac{1}{2} + \frac{1}{2}\log 2}}$$

(3) $f = \min\{\frac{\pi}{2}, x\}$

kos. $\sum$



$$V(f, [a, \pi]) = 2 \cdot \frac{\pi}{2} = \frac{\pi}{2}$$

$$a_0 = \frac{2}{\pi}\left(\int_0^{\pi/2} x + \int_{\pi/2}^{\pi} \frac{\pi}{2}\right) = \frac{2}{\pi}\left(\left[\frac{x^2}{2}\right]_0^{\pi/2} + \frac{\pi}{2}\cdot\frac{\pi}{2}\right) = \frac{2}{\pi}\left(\frac{1}{2}\cdot\frac{\pi^2}{4} + \frac{\pi^2}{4}\right) = \frac{3}{2}\frac{\pi^2}{4}\cdot\frac{2}{\pi} = \underline{\underline{\frac{3\pi}{4}}}$$

$$a_n = \frac{2}{\pi}\left(\int_0^{\pi/2} x\cos(nx) + \int_{\pi/2}^{\pi} \frac{\pi}{2}\cos nx\right) = \frac{2}{\pi}\left(\left[\frac{x}{n}\sin(nx) + \frac{1}{n^2}\cos nx\right]_0^{\pi/2} + \left[\frac{\pi}{2}\,\frac{\sin(nx)}{n}\right]_{\pi/2}^{\pi}\right)$$

$n \in \mathbb{N}$

$$\int \underset{u}{x}\,\underset{v'}{\cos(nx)} = x\,\frac{\sin(nx)}{n} - \frac{1}{n}\int \sin(nx) = \frac{x}{n}\sin(nx) + \frac{1}{n^2}\cos(nx)$$

$u' = 1 \qquad v = \frac{\sin(nx)}{n}$

$$a_n = \frac{2}{\pi}\left(\frac{\pi}{2n} \sin\left(n\frac{\pi}{2}\right) + \frac{1}{n^2}\cos n\frac{\pi}{2} - \frac{1}{n^2} - \frac{\pi}{2}\,\frac{\sin\left(n\frac{\pi}{2}\right)}{n}\right)$$

$$= \frac{2}{\pi}\frac{1}{n^2}\left(\cos\left(n\frac{\pi}{2}\right) - 1\right)$$

$$S_f = \frac{3\pi}{8} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left(\cos\left(n\frac{\pi}{2}\right) - 1\right)$$

f je spoj na $\mathbb{R}$, je BV.

Z J-D: $S_f \rightrightarrows f$ na $\mathbb{R}$

($S_f \to f$, f spoj., dále $S_f \overset{loc}{\rightrightarrows} f$ na $(-2\pi, 2\pi)$; z definice lok. stejnom. konv. tedy $S_f \rightrightarrows f$ na $[-\pi, \pi]$; z periodicity $S_f \rightrightarrows f$ na $\mathbb{R}$