

$$x^2 + y^2 = 1$$



$$y = \pm \sqrt{1-x^2}$$

$$x = \pm \sqrt{1-y^2}$$

$$0 = x^2 + y^2 - 1 - x^{2/3} y$$

$$F(x, y) = x^2 + y^2 - 1 - y \sqrt[3]{x^2} \quad \rightarrow a = [1, 1]$$

$$F \in C^1(G) \quad G = B((1, 1), 1/4) \quad (\text{pozdor nem } x=0)$$

$$F(1, 1) = 1 + 1 - 1 - 1 = 0$$

$$\frac{\partial F}{\partial y} = 2y - \sqrt[3]{x^2} \quad \frac{\partial F}{\partial y}(1, 1) = 2 - 1 = 1 \neq 0$$

$$\frac{\partial F}{\partial x} = 2x - y \frac{2}{3} \frac{1}{\sqrt[3]{x}} \quad \frac{\partial F}{\partial x}(1, 1) = 2 - \frac{2}{3} = \frac{4}{3}$$

$$y' = - \frac{2x - y \frac{2}{3} \frac{1}{\sqrt[3]{x}}}{2y - \sqrt[3]{x^2}}$$

de vzorec-ku:

$$y'(1) = - \frac{4/3}{1} = - \frac{4}{3}$$

de řetězového pravidla

$$0 = x^2 + (y(x))^2 - 1 - x^{2/3} (y(x))$$

$$\frac{d}{dx}: \quad 0 = 2x + 2y \cdot y' - \frac{2}{3} x^{-1/3} y - x^{2/3} y'$$

$$y'(2y - x^{2/3}) = -2x + \frac{2}{3} x^{1/3} y$$

$$y' = \frac{-2x + \frac{2}{3} y \sqrt[3]{x}}{2y - x^{2/3}}$$

$$\begin{matrix} x_0 & y_0 \\ \swarrow & \searrow \\ (1, 1) \end{matrix}$$

$$y'(1) = \frac{-2 + \frac{2}{3}}{2 - 1} = -\frac{4}{3}$$

2. dce:

$$0 = 2x + 2y \cdot y' - \frac{2}{3} x^{-1/3} y - x^{2/3} y'$$

$\frac{d}{dx}$:

$$0 = 2 + 2y'y' + 2yy'' - \frac{2}{3}(-\frac{1}{3})x^{-4/3}y - \frac{2}{3}x^{-1/3} - \frac{2}{3}x^{-1/3} - x^{2/3}y''$$

$v(1,1)$

$$y'(1) = -\frac{4}{3}$$

$$0 = 2 + 2 \cdot (-\frac{4}{3})^2 + 2y'' + \frac{2}{9} + \frac{2}{3} \cdot \frac{4}{3} + \frac{2}{3} \cdot \frac{4}{3} - y''$$
$$-\frac{68}{9} = y''$$