① $$\lim_{x\to0} \frac{\sqrt{e^{x^2}+3}-2}{x^2} = \lim_{x\to0} \frac{e^{x^2}+3-4}{x^2}\cdot\frac{1}{\sqrt{e^{x^2}+3}+2}$$

$$= \lim_{x\to0}\frac{e^{x^2}-1}{x^2}\cdot\frac{1}{\sqrt{e^{x^2}+3}+2} \overset{AL}{=} 1\cdot\frac{1}{2+2} = \frac{1}{4}$$

VOLSF1 $f(y)=\frac{e^y-1}{y}$ $\lim_{y\to0}\frac{e^y-1}{y}=1$

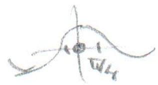
$g(x)=x^2$ $\lim_{x\to0}x^2=0$ (P) $x^2\neq0$ na $P(0,1)$

$\lim_{x\to0}\sqrt{e^{x^2}+3}=\sqrt{e^0+3}=2$ protože $\sqrt{e^{x^2}+3}$ je spoj. (složení a součet spoj.)

② $$\lim_{x\to0}(\cos x)^{\cot x} = e^{\cot x\cdot\log(\cos x)} = e^0 = 1$$

$$\lim_{x\to0} \underbrace{\cos x}_{\to1}\, \underbrace{\frac{x}{\sin x}}_{\to1}\cdot\underbrace{\frac{\log(\cos x)}{\cos x-1}}_{\to1}\cdot\underbrace{\frac{(\cos x-1)}{x^2}}_{-\frac{1-\cos x}{x^2}\to-\frac{1}{2}}\cdot\underbrace{\frac{x^2}{x}}_{x\to0} \overset{AL}{=} 1\cdot1\cdot-\frac{1}{2}\cdot0$$

spoj.



VOLSF1 $f=\frac{\log y}{y-1}$ $\lim_{y\to1}\frac{\log y}{y-1}=1$

$g=\cos x-1$ $\lim_{x\to0}\cos x-1=0$ (P) $\cos x-1\neq0$ na $P(0,\pi/4)$

VOLSF2 $f=e^y$ $\lim_{y\to0}e^y=1$ (S) e^y spoj. v 0

$g=\cot x\cdot\log(\cos x)$ $\lim g=0$

③ $$f(x)=\frac{\sqrt[3]{x^4+x}}{\sin(\log x)+2}$$ D_f: $x>0$

$$f' = \frac{\frac{1}{3}(x^4+x)^{-2/3}(4x^3+1)(\sin(\log x)+2) - \sqrt[3]{x^4+x}\cdot\left(\cos(\log x)\cdot\frac{1}{x}\right)}{(\sin(\log x)+2)^2}$$

$D_{f'}$: $x>0$

(4) f period, nekonst.

Předpokládejme, že $\lim_{x \to \infty} f(x) = A \in \mathbb{R}^*$

Nechť p značí periodu.

Dále, f je nekonstantní, tedy $\exists\, a, b \in \mathbb{R}$: $f(a) \neq f(b)$.

Pak pro posl. $a_n = f(a + np)$ a $b_n = f(b + np)$

máme

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} f(a + np) = \lim_{n \to \infty} f(a) = f(a)$$

$$\lim_{n \to \infty} b_n = f(b)$$

Ale $f(b) \neq f(a)$, což je spor s Heineho větou