① $f_n = \dfrac{\sin(nx)}{n^2+x^4}$

(a) fix $x \in \mathbb{R}$: $\lim\limits_{n\to\infty} \dfrac{\sin(nx)}{n^2+x^4} = \lim \dfrac{1}{n^2} \cdot \underbrace{\dfrac{\sin(nx)}{1+\frac{x^4}{n^2}}}_{\text{om a uiz.}} = 0$

$f \equiv 0$

(b) fix n: $\sigma_n = \sup \left\{ \left| \dfrac{\sin nx}{n^2+x^4} \right|, x \in \mathbb{R} \right\}$

$$\frac{|\sin nx|}{n^2+x^4} \le \frac{1}{n^2+x^4} \le \frac{1}{n^2}$$

$\sigma_n \le \frac{1}{n^2} \Rightarrow \lim\limits_{n\to\infty} \sigma_n = 0 \rightarrow f_n \rightrightarrows 0$ na $\mathbb{R}$

② $f_n = \dfrac{n^2 x}{1+n^4x^2}$ $x \in (0,\infty)$

(a) fix $x \in (0,\infty)$: $\lim\limits_{n\to\infty} \dfrac{n^2x}{1+n^4x^2} = \lim \dfrac{n^2}{n^4} \cdot \dfrac{x}{\frac{1}{n^4}+x^2} \stackrel{AL}{=} 0 \cdot \dfrac{x}{0+x^2} = 0$

$f \equiv 0$

(b) fix n: $\sigma_n = \sup \left\{ \left| \dfrac{n^2x}{1+n^4x^2} \right|, x \in \mathbb{R} \right\}$

$f_n' = \dfrac{n^2(1+n^4x^2) - n^2x \cdot 2xn^4}{(1+n^4x^2)^2}$ $\qquad n^2+n^6x^2-2x^2n^6 = 0$

$n^2 = x^2n^6$

$f_n\left(\frac{1}{n^2}\right) = \dfrac{1}{1+1} = \dfrac{1}{2}$ $\qquad \frac{1}{n^4} = x^2$

$\frac{1}{n^2} = x$

$\sigma_n \ge 1/2 \Rightarrow \lim \sigma_n \ne 0 \Rightarrow f_n \not\rightrightarrows 0$ na $(0,\infty)$

③ $\sum x^2 e^{-n^2x^2}$ $x \in [0,\infty)$

(a) fix $x \in [0,\infty)$: $x = 0$ $\sum 0$ K

$x \ne 0$ $x^2 \sum \left(e^{-x^2}\right)^{n^2} \rightarrow$ geom. řada (resp. vybr. členy geom. $\sum$) K

(b) fix n, $\sigma_n = \sup \{ x^2 e^{-n^2x^2}, x \in [0,\infty) \}$

$f_n' = 2x e^{-n^2x^2} + x^2 e^{-n^2x^2} \cdot 2x(-n^2)$ $\qquad 1-n^2x^2 = 0 \quad x^2 = \frac{1}{n^2} \quad x = \frac{1}{n}$

$f(1/n) = \frac{1}{n^2} e^{-1}$

kraje $f_n(0) = 0$

$\lim\limits_{x\to\infty} f_n = 0$ ← klesá

$\Bigg\}$ $\sigma_n = \dfrac{e^{-1}}{n^2}$ $\quad \sum \dfrac{1}{en^2}$ K $\Rightarrow \sum f_n \rightrightarrows$ na $[0,\infty)$