(1) $\sum \underbrace{\frac{2^n + (-1)^n n}{3^n + (-1)^n n}}_{a_n > 0}$

d'Al

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{\dfrac{2\cdot 2^n + (-1)^{n+1}(n+1)}{3\cdot 3^n + (-1)^{n+1}(n+1)}}{\dfrac{2^n + (-1)^n n}{3^n + (-1)^n n}} =$$

$$= \lim \frac{\left(\frac{2}{3}\right)^n}{\left(\frac{2}{3}\right)^n} \cdot \frac{\dfrac{2 + \frac{(-1)^{n+1}(n+1)}{2^n}}{3 + \frac{(-1)^{n+1}(n+1)}{3^n}}}{\dfrac{1 + \frac{(-1)^n n}{2^n}}{1 + \frac{(-1)^n}{3^n}}} \overset{AL}{=} \frac{\frac{2+0}{3+0}}{\frac{1+0}{1+0}} = \frac{2}{3} < 1$$

z dův. lim:

$$\lim_{x\to\infty} \frac{x}{3^x} \overset{LH}{\underset{\frac{\infty}{\infty}}{=}} \lim \frac{1}{3^x \log 3} =$$

Heine $x_n = n \to \infty$, $n \neq \infty$

$$\lim_{n\to\infty} \frac{n}{3^n} = 0$$

pak z om a uniž

$$(-1)^n \cdot \frac{n}{3^n} = 0$$

ostatní analogicky

$\Rightarrow \sum a_n$ k dle d'Al. kritéria

(2) $\sum \underbrace{\frac{1 - \cos\frac{1}{\sqrt{n}}}{\sqrt{n+3}}}_{a_n > 0}$

LSK s $b_n = \dfrac{\frac{1}{(\sqrt{n})^2}}{\sqrt{n}} = \dfrac{1}{n\sqrt{n}} = \dfrac{1}{n^{3/2}} > 0$ $\quad \sum \frac{1}{n^{3/2}}$ k

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim \frac{1-\cos\frac{1}{\sqrt{n}}}{\frac{1}{n}} \cdot \frac{\sqrt{n}}{\sqrt{n+3}} = \frac{1}{2}\cdot 1 = \frac{1}{2} \in (0,\infty)$$

Tedy $\sum a_n$ k z LSK

Limity: $x_n = \frac{1}{\sqrt{n}}$, $\frac{1}{\sqrt{n}} \to 0$, $\frac{1}{\sqrt{n}} \neq 0$

Heine

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \quad \text{tedy i} \quad \lim_{n\to\infty} \frac{1-\cos\frac{1}{\sqrt{n}}}{1/n} = 1/2$$

$$\lim \frac{\sqrt{n}}{\sqrt{n+3}} = \lim \frac{1}{\sqrt{1+3/n}} = 1$$

$\sqrt{\underbrace{1+3/n}_{\to 1+0}}$ $\quad$ Heine + VOLSF (s)

(3) $\sum \underbrace{\cos(n\pi)}_{(-1)^n} \cdot \underbrace{\frac{1}{n+\frac{1}{n^2}}}_{b_n > 0}$

$$\lim \frac{1}{n+1/n^2} = \frac{1}{\infty + 0} = 0$$

bn je nerostoucí:

$$\frac{1}{n+1/n^2} \geq \frac{1}{n+1+\frac{1}{(n+1)^2}}$$

$$\cancel{n}+1 + \frac{1}{(n+1)^2} \geq \cancel{n} + \frac{1}{n^2} \quad \checkmark$$

$$1 + \frac{1}{(n+1)^2} \geq 1 + 0 \geq \frac{1}{n^2}$$

tedy $\sum$ z Leibnize konverguje

(4) (a) Neplatí:
$$a_n = \begin{cases} 0 & n \text{ liché} \\ \frac{1}{n^2} & n \text{ sudé} \end{cases}$$

$$b_n = \begin{cases} n & n \text{ liché} \\ \frac{1}{n^2} & n \text{ sudé} \end{cases}$$

Pak $\limsup \frac{a_n}{b_n} = 1$, $\sum a_n \leq \sum \frac{1}{n^2}$ konv., ale $\sum b_n \geq \sum 2n+1$ div.

(b) Neplatí: $b_n = (-1)^n \frac{1}{n}$ $\quad \sum b_n$ konv.

$a_n = \frac{1}{n}$ $\quad \sum a_n$ div.

ale $\limsup \frac{a_n}{b_n} = 1$