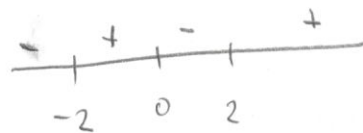


①



$$f = |x(x^2 - 4)|$$

$$= \begin{cases} x^3 - 4x & x \in [2, \infty) \\ -x^3 + 4x & x \in (0, 2) \\ x^3 - 4x & x \in [-2, 0) \\ -x^3 + 4x & x \in (-\infty, -2) \end{cases}$$

f je spoj na $\mathbb{R} \rightarrow f$ má na \mathbb{R} PF

$$F = \begin{cases} \frac{x^4}{4} - 2x^2 + c_1 & x \in (2, \infty) \\ -\frac{x^4}{4} + 2x^2 + c_2 & x \in (0, 2) \\ \frac{x^4}{4} - 2x^2 + c_3 & x \in (-2, 0) \\ -\frac{x^4}{4} + 2x^2 + c_4 & x \in (-\infty, -2) \end{cases}$$

$$\lim_{x \rightarrow 2^-} -\frac{x^4}{4} + 2x^2 + c_2 = \lim_{x \rightarrow 2^+} \frac{x^4}{4} - 2x^2 + c_1$$

$$-4 + c_2 = -4 + c_1$$

$$\boxed{c_2 = c_1 - 8}$$

$$\lim_{x \rightarrow 0^-} \frac{x^4}{4} - 2x^2 + c_3 = \lim_{x \rightarrow 0^+} -\frac{x^4}{4} + 2x^2 + c_2$$

$$\boxed{c_3 = c_2 = c_1 - 8}$$

$$\lim_{x \rightarrow -2^-} -\frac{x^4}{4} + 2x^2 + c_4 = \lim_{x \rightarrow -2^+} \frac{x^4}{4} - 2x^2 + c_3$$

$$-4 + c_4 = -4 + c_3$$

$$\boxed{c_4 = c_3 - 8 = c_1 - 16}$$

závěr

$$F = \begin{cases} \frac{x^4}{4} - 2x^2 + c_1 & x \in [2, \infty) \\ -\frac{x^4}{4} + 2x^2 + c_1 - 8 & x \in (0, 2) \\ \frac{x^4}{4} - 2x^2 + c_1 - 8 & x \in [-2, 0) \\ -\frac{x^4}{4} + 2x^2 + c_1 - 16 & x \in (-\infty, -2) \end{cases}$$

④ f má PF na \mathbb{R} , g je polynom. Pak \exists PF k fg na \mathbb{R}

pot parťas: $\int fg = Fg - \int Fg'$

F je spoj na \mathbb{R} (má vlastní dui)

\downarrow
spoj na \mathbb{R}

tedy Fg' spoj \rightarrow tedy $\int Fg'$

tedy: tvrzení je pravdivé

$$\textcircled{2} \int \frac{2x^2+3x+6}{(x+2)(x^2+4)} dx = \int \frac{A}{x+2} + \frac{Bx+C}{x^2+4} dx$$

st P < st Q \downarrow
 nema' zovrac

$$2x^2+3x+6 = A(x^2+4) + (Bx+C)(x+2)$$

$$x = -2 \quad 8 - 6 + 6 = 8A \quad A = 1$$

$$x = 0 \quad 6 = 4 + 2C \quad C = 1$$

$$x = 1 \quad 11 = 5 + 3B + 3 \quad B = 1$$

$$\int \frac{1}{x+2} \stackrel{C}{=} \log |x+2|$$

$$\int \frac{x+1}{x^2+4} dx = \frac{1}{2} \int \frac{2x}{x^2+4} dx + \int \frac{1}{x^2+4} dx \stackrel{C}{=} \frac{1}{2} \log(x^2+4) + \frac{1}{2} \arctan \frac{x}{2}$$

$$\frac{1}{2} \log(x^2+4) \quad \frac{1}{4} \int \frac{1}{\left(\frac{x}{2}\right)^2+1} dx = \frac{1}{4} \cdot 2 \arctan \frac{x}{2}$$

Dobromady

$$\int \stackrel{C}{=} \log |x+2| + \frac{1}{2} \log(x^2+4) + \frac{1}{2} \arctan \frac{x}{2} \quad x \in (-\infty, -2), (-2, \infty)$$

$$\textcircled{3} \int_0^1 \frac{3 - \arctan x}{1+x^2} \sin(\arctan x) dx = \int_0^{\pi/4} (3-y) \sin y dy = \left[-(3-y) \cos y \right]_0^{\pi/4} - \int_0^{\pi/4} \cos y dy$$

$$y = \arctan x$$

$$dy = \frac{1}{1+x^2} dx$$

x	0	1
y	0	$\pi/4$

$$u' = -1 \quad v = -\cos y$$

$$= \left[-(3-y) \cos y \right]_0^{\pi/4} - \left[\sin y \right]_0^{\pi/4} = -\left(3 - \frac{\pi}{4}\right) \frac{\sqrt{2}}{2} + 3 \cdot 1 - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \left(\frac{\pi}{4} - 1 - 3\right) + 3$$

$$= \frac{\sqrt{2}}{2} \left(\frac{\pi}{4} - 4\right) + 3$$