− + − +

−2 0 2

① $f = |x(x^2-4)|$

$$= \begin{cases} x^3-4x & x\in[2,\infty) \\ -x^3+4x & x\in[0,2) \\ x^3-4x & x\in[-2,0) \\ -x^3+4x & x\in(-\infty,-2) \end{cases}$$

f je spoj na $\mathbb{R}$ → f má na $\mathbb{R}$ PF

$$F = \begin{cases} \frac{x^4}{4}-2x^2+c_1 & x\in(2,\infty) \\ -\frac{x^4}{4}+2x^2+c_2 & x\in(0,2) \\ \frac{x^4}{4}-2x^2+c_3 & x\in(-2,0) \\ -\frac{x^4}{4}+2x^2+c_4 & x\in(-\infty,-2) \end{cases}$$

$$\lim_{x\to 2-} -\frac{x^4}{4}+2x^2+c_2 = \lim_{x\to 2+} \frac{x^4}{4}-2x^2+c_1$$

$$4+c_2 = -4+c_1$$

$$\boxed{c_2 = c_1-8}$$

$$\lim_{x\to 0-} \frac{x^4}{4}-2x^2+c_3 = \lim_{x\to 0+} -\frac{x^4}{4}+2x^2+c_2$$

$$\boxed{c_3 = c_2 = c_1-8}$$

$$\lim_{x\to -2-} -\frac{x^4}{4}+2x^2+c_4 = \lim_{x\to -2+} \frac{x^4}{4}-2x^2+c_3$$

$$4+c_4 = -4+c_3$$

$$\boxed{c_4 = c_3-8 = c_1-16}$$

závěr $$F = \begin{cases} \frac{x^4}{4}-2x^2+c_1 & x\in[2,\infty) \\ -\frac{x^4}{4}+2x^2+c_1-8 & x\in[0,2) \\ \frac{x^4}{4}-2x^2+c_1-8 & x\in[-2,0) \\ -\frac{x^4}{4}+2x^2+c_1-16 & x\in(-\infty,-2) \end{cases}$$

④ f má PF na $\mathbb{R}$, g je polynom. Pak $\exists$ PF k fg na $\mathbb{R}$

Per partes: $\int fg = Fg - \int Fg'$ F je spojitá na $\mathbb{R}$ (má vlastní der)

g' spojitá na $\mathbb{R}$

tedy Fg' spoj → tedy $\int \exists$

tedy: Tvrzení je pravdivé

(2) $\int \frac{2x^2+3x+6}{(x+2)(x^2+4)}\,dx = \int \frac{\overset{A}{1}}{x+2} + \frac{\overset{Bx+C}{x+1}}{x^2+4}\,dx$

st P < st Q ✓ nemá kořen

$$2x^2+3x+6 = A(x^2+4) + (Bx+C)(x+2)$$

$x=-2$ $\quad 8-6+6 = 8A \quad A = 1$

$x=0$ $\quad 6 = 4+2C \quad C = 1$

$x=1$ $\quad 11 = 5 + 3B + 3 \quad B = 1$

$$\int \frac{1}{x+2} \overset{c}{=} \log|x+2|$$

$$\int \frac{x+1}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx \overset{c}{=} \frac{1}{2}\log(x^2+4) + \frac{1}{2}\arctan\frac{x}{2}$$

$$\frac{1}{2}\log(x^2+4) \qquad \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{4}\cdot 2\arctan\frac{x}{2}$$

Dohromady

$$\int \overset{c}{=} \log|x+2| + \frac{1}{2}\log(x^2+4) + \frac{1}{2}\arctan\frac{x}{2} \qquad x \in (-\infty,-2),\ (-2,\infty)$$

(3) $\int_0^1 \frac{3-\arctan x}{1+x^2} \sin(\arctan x)\,dx = \int_0^{\pi/4} \underset{u}{(3-y)} \underset{v'}{\sin y}\,dy = \left[-(3-y)\cos y\right]_0^{\pi/4} - \int_0^{\pi/4} \cos y$

$y = \arctan x$

$dy = \frac{1}{1+x^2}\,dx$

x	0	1
y	0	$\pi/4$

$u' = -1 \qquad v = -\cos y$

$$= \left[-(3-y)\cos y\right]_0^{\pi/4} - \left[\sin y\right]_0^{\pi/4} = -\left(3-\frac{\pi}{4}\right)\frac{\sqrt{2}}{2} + 3\cdot 1 - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(\frac{\pi}{4} - 1 - 3\right) + 3$$

$$= \underline{\underline{\frac{\sqrt{2}}{2}\left(\frac{\pi}{4} - 4\right) + 3}}$$