(1) $\int_0^\infty \frac{(\arctan bx)^2 - (\arctan ax)^2}{x}\,dx = F(a,b)$

Nechť $0<a\le b$

$= \int_0^\infty \left[\frac{(\arctan yx)^2}{x}\right]_a^b dx = \int_0^\infty\int_a^b \frac{1}{x}\,\frac{2\arctan yx}{1+(yx)^2}\cdot x\,dy\,dx$

Fub $= \int_a^b\int_0^\infty \overbrace{\frac{2\arctan yx}{1+(yx)^2}}^{f(x,y)}\,dx\,dy = \int_a^b \left[\frac{1}{y}(\arctan yx)^2\right]_0^\infty dy =$ ← $0<a\le b$

(4) $= \int_a^b \frac{1}{y}\,\frac{\pi^2}{4}\,dy = \frac{\pi^2}{4}\left[\ln|y|\right]_a^b = \frac{\pi^2}{4}\ln\left|\frac{b}{a}\right|$

$0<a\le b$

z důk. Fub. $\frac{2\arctan yx}{1+(yx)^2} \ge 0$ na $(0,\infty)\times(a,b)$

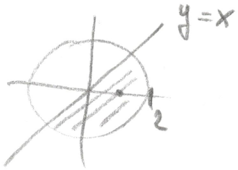
(2) $0<a<b$

Rozbor případů: $a=b$ $F(a,a)=0$ zřejmě

$a=0$ nebo $b=0$
$b\neq 0$ $a\neq 0$ → $\int_0^\infty \frac{\arctan(bx)^2}{x} = \infty$ LSK s $\frac{1}{x}$ u $\infty$

$F(a,-b) = F(a,b) = F(-a,b) = F(-a,-b)$ F je sudá v a i b.

(2) $\int_M xy\,d\lambda$ M: $x^2+y^2<4$; $y<x$




(2) $x = r\cos\alpha$ $r\in(0,2)$
$y = r\sin\alpha$ $\alpha\in(-\frac{3}{4}\pi, \pi/4)$

(1) $\int_{-\frac{3}{4}\pi}^{\frac{\pi}{4}}\int_0^2 \underbrace{r^2\cos\alpha\sin\alpha\, r}_{\text{spoj. ve tod.}}\,dr\,d\alpha = \int_{-\frac{3}{4}\pi}^{\pi/4} \cos\alpha\sin\alpha\left[\frac{1}{4}r^4\right]_0^2 d\alpha = 4\left[\frac{1}{2}\sin^2\alpha\right]_{-\frac{3}{4}\pi}^{\pi/4}$

(3)

$= 2\left(\frac{2}{4}-\frac{2}{4}\right) = \underline{0}$