2A

(1) $$\lim_{n\to\infty} \frac{2n - n^3 + 4/n}{n^2 + 3n^3 - \frac{5}{\sqrt[3]{n}}} \overset{(2+3)}{=} \lim_{n\to\infty} \frac{n^3}{n^3} \cdot \frac{\frac{2}{n^2} - 1 + \frac{4}{n^4}}{\frac{1}{n} + 3 - \frac{5}{\sqrt[3]{n}\cdot n^3}} \overset{AL}{=} \frac{0 - 1 + 0}{0 + 3 - 0} = -\frac{1}{3}$$

(1) (3) (1)

(2) $$\lim_{n\to\infty} \sqrt{2n^2+3}\left(\sqrt{n^2+2} - \sqrt{n^2-4}\right)$$

(1 + 1 + 2)

$$= \lim_{n\to\infty} \frac{\sqrt{2n^2+3}\,(n^2+2-n^2+4)}{\sqrt{n^2+2}+\sqrt{n^2-4}} = \lim_{n\to\infty} \frac{6\sqrt{n^2}}{\sqrt{n^2}} \cdot \frac{\sqrt{2+3/n^2}}{\sqrt{1+2/n^2}+\sqrt{1-4/n^2}}$$

(1) (2)

(1)

$$\overset{AL}{=} \quad 6 \cdot \frac{\sqrt{2+0}}{\sqrt{1+0}+\sqrt{1-0}} = 3\sqrt{2}$$

(1)

věta o odmocnině

(3) $$\lim_{n\to\infty} \sqrt[n]{2^n+n+3+\log n} = 2$$

(5 + 1)

$$2 = \sqrt[n]{2^n+0+0+0} \le \sqrt[n]{2^n+n+3+\log n} \overset{(*)}{\le} \sqrt[n]{2^n+2^n+2^n+2^n} = \sqrt[n]{4\cdot 2^n} = \sqrt[n]{4}\cdot 2$$

$\downarrow$ 2 $\qquad\qquad$ $\downarrow$ $1\cdot 2 = 2$

(1) ze 2 policajtů i $\lim \sqrt[n]{2^n+n+3+\log n} = 2$

(3)

(*) platí od jistého n_0: $\forall n \ge n_0$ ze škály

$n \le 2^n$ to jest $\frac{n}{2^n} \le 1$ $\qquad$ $\lim \frac{n}{2^n} = 0$ tedy $\exists n_1\ \forall n \ge n_1$ $\frac{n}{2^n} \le 1$

analogicky $\quad 3 \le 2^n \quad \frac{3}{2^n} \le 1 \quad \lim \frac{3}{2^n} = 0 \quad \exists n_2\ \forall n \ge n_2 \quad \frac{3}{2^n} \le 1$

$\log n \le 2^n \quad \frac{\log n}{2^n} \le 1 \quad \lim \frac{\log n}{2^n} = 0 \quad \exists n_3\ \forall n \ge n_3 \quad \frac{\log n}{2^n} \le 1$

$n_0 = \max\{n_1, n_2, n_3\}$

① $\lim\limits_{n\to\infty} \dfrac{n+\sqrt{n}-\frac{5}{n^4}}{\sqrt[4]{n^3}+3n+6}$ (2+3) $= \lim \dfrac{n}{n} \cdot \dfrac{1+\frac{1}{\sqrt{n}}-\frac{5}{n^5}}{\frac{1}{\sqrt[4]{n}}+3+\frac{6}{n}} \overset{\text{VOAL}}{=} \dfrac{1+0-0}{0+3+0} = \dfrac{1}{3}$ ① ③ ①

(1+1+2)

② $\lim\limits_{n\to\infty} \sqrt{4n}\left(\sqrt{n^3-1}-\sqrt{n^3-n}\right) = \lim \dfrac{\sqrt{4n}\left(n^3-1-(n^3-n)\right)}{\sqrt{n^3-1}+\sqrt{n^3-n}}$

(1+3)

$= \lim \dfrac{\sqrt{4n}\,(n-1)}{\sqrt{n^3-1}+\sqrt{n^3-n}} = \lim \dfrac{n\sqrt{n}}{n\sqrt{n}} \cdot \dfrac{\sqrt{4}-\frac{2}{n}}{\sqrt{1-\frac{1}{n^3}}+\sqrt{1-\frac{1}{n^2}}}$

① ①

$\overset{\text{VOAL}}{\underset{\text{Vo}\sqrt{\ }}{=}} \dfrac{\sqrt{4}-0}{\sqrt{1-0}+\sqrt{1-0}} = \dfrac{\sqrt{4}}{2} = \underline{1}$

③ $\lim \sqrt[n]{\frac{1}{2^n}+n+4^n+\sqrt{n}} = 4$

(5+1)

2 policajti

$4 = \sqrt[n]{4^n} \le \sqrt[n]{4^n+n+\sqrt{n}+\frac{1}{2^n}} \le \sqrt[n]{4\cdot 4^n} = \sqrt[n]{4}\cdot 4$

① ↓AL

③

$1\cdot 4$

$\exists n_0 : \forall n \ge n_0$ $\quad n \le 4^n$ protože $\lim \frac{n}{4^n} = 0$

$-\,11\,-$ $\quad \sqrt{n} \le n \le 4^n$

$\frac{1}{2^n} \le 4^n \quad \forall n \in \mathbb{N}$

(1) $\lim\limits_{n\to\infty} \dfrac{n^4 - \sqrt{2n}}{n^4 + 3n^2 - 5/n} = \lim\limits_{n\to\infty} \dfrac{\cancel{n^4}}{\cancel{n^4}} \cdot \dfrac{1 - \frac{\sqrt{2}}{n^3\sqrt{n}}}{1 + \frac{3}{n^2} - \frac{5}{n^5}} \overset{\text{VOAL}}{=} \dfrac{1-0}{1+0-0} = 1$

(2+3) $\to 0$ (1) (3) (1)

$\frac{3}{n^2} \to 0$, $\frac{5}{n^5} \to 0$

(1+1+2)

(2) $\lim\limits_{n\to\infty} (n-2)\left(\sqrt{n^2-1} - n\right) \cdot \dfrac{\sqrt{n^2-1}+n}{\sqrt{n^2-1}+n} = \lim\limits_{n\to\infty} \dfrac{(n-2)(n^2-1-n^2)}{\sqrt{n^2-1}+n}$

(1) (3) (1) (1)

$= \lim\limits_{n\to\infty} \dfrac{-1\cdot n}{n} \cdot \dfrac{1-2/n}{\sqrt{1-1/n^2}+1} \overset{\text{VOAL}}{\underset{\text{Vo}\sqrt{\ }}{=}} -1 \cdot \dfrac{1-0}{\sqrt{1-0}+1} = -\dfrac{1}{2}$

(3) $\lim\limits_{n\to\infty} \sqrt[n]{5^n + 2 + \sin n} = 5$

pol.
(5) (2) lim
(1) pol.
(2) zaver

2 policajti

$\sqrt[n]{5^n} \le \sqrt[n]{5^n+1} \le \sqrt[n]{5^n+2+\sin n} \le \sqrt[n]{5^n+3} \le \sqrt[n]{2\cdot 5^n}$

$\sqrt[n]{5^n} = 5$

$\sqrt[n]{2\cdot 5^n} = \sqrt[n]{2}\cdot 5 \xrightarrow{\text{AL}} 1\cdot 5$