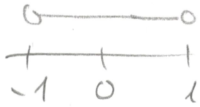
① $\sum_{n=1}^{\infty} (n^2-1)\, x^n$

(a) $\lim_{n\to\infty} \sqrt[n]{n^2-1} = 1$



② $\sum (n^2-1)\,(\pm 1)^n$ Div

(b) $f(x) = \sum (n^2-1)\, x^n$

② $F = \sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1}(n-1)(n+1) = \underbrace{\sum_{n=1}^{\infty} (n-1)\, x^{n-2} \cdot x^3}_{g(x)}$

② $G = \sum_{n=1}^{\infty} x^{n-1} = \frac{1}{1-x} = (1-x)^{-1}$

① $g = \frac{+1}{(1-x)^2}$

② $F = \frac{x^3}{(1-x)^2} \to f = \frac{3x^2(1-x)^2 + x^3 \cdot 2(1-x)}{(1-x)^4} = \frac{3x^2(1-x) + 2x^3}{(1-x)^3}$

$= \frac{3x^2 - x^3}{(1-x)^3}$

② $x = r\cos\alpha$

4 $y = y$ $\qquad r\sin\alpha \geq 0 \to \sin\alpha \geq 0 \qquad \alpha \in (0,\pi)$

$z = r\sin\alpha \qquad r \in (0,3) \qquad y \in (-1,2)$

$\int_0^{\pi}\int_{-1}^{2}\int_0^{3} y \cdot r \cdot r\, dr\, dy\, d\alpha = \iint y \frac{1}{3}[r^3]_0^3\, dy\, d\alpha = 9\int \left[\frac{y^2}{2}\right]_{-1}^{2} d\alpha =$

2

3 $9\int_0^{\pi} \left(2 - \frac{1}{2}\right) d\alpha = \frac{27}{2}\pi$

1

(3) $\int_0^\infty \frac{\arctan x}{\sqrt{1+x^3}}\,dx$ $\qquad$ f spoj na $[0,\infty)$ $\quad f \geq 0$

(1)

(1) $= \int_0^1 + \int_1^\infty$

$\downarrow$

(1) spoj na kpt $\to$ konv.

$\to \infty$ LSK s $g(x) = \frac{1}{\sqrt{x^3}}$ $\qquad \int_1^\infty x^{-3/2}$ K $\qquad$ (2)

(1) g spoj na $[0,\infty)$

$g > 0$

(3) $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim \frac{\arctan x}{\sqrt{1+x^3}} \cdot \sqrt{x^3} = \frac{\pi}{2} \in (0,\infty)$

$\to \int_0^\infty f$ K $\quad$ (dakako AK)

(1)