① $x^3 + y^7 = e^{xy^2} - \sin y$ $\qquad [1,0]$

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1 • $F(x,y) = x^3 + y^7 - e^{xy^2} + \sin y$ $\qquad F \in C^\infty(\mathbb{R}^2)$

1 • $F(1,0) = 1 + 0 - e^0 + \sin 0 = 0$

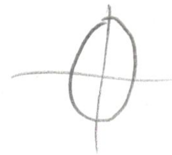
2 • $\frac{\partial F}{\partial y} = 7y^6 - e^{xy^2} \cdot 2xy + \cos y$ $\qquad$ v $[1,0]$: $7 \cdot 0 - e^0 \cdot 0 + \cos 0 = 1 \neq 0$

Pak F určuje na jistém okolí $[1,0]$ impl. zadanou funkci φ.

4

1 $\qquad \varphi'(1) = - \frac{3x^2 - e^{xy^2} \cdot y^2}{7y^6 - e^{xy^2} \cdot 2xy + \cos y} \Big|_{[1,0]} = - \frac{3-0}{1} = \underline{\underline{-3}}$

② $f = xy$ $\qquad M: \; 4x^2 + y^2 = 8$



3 • f je spoj. na $\mathbb{R}^2$ (polynom) $\qquad$ M elipsa.

omezená: $\quad x^2 \le 8 \quad y^2 \le 8$

uzavřená: $\quad g(x,y) = 4x^2 + y^2 - 8$ spoj.

$M = g^{-1}(\{0\})$ vzor uz. při spoj. zobr. → uz.

∃ f na M nabývá extrému

• Lagrange

1 ∘ $\nabla g = (8x, 2y) = (0,0) \iff x = 0, y = 0 \notin M$

1 ∘ $\nabla f + \lambda \nabla g = (0,0)$

$y + \lambda 8x = 0$ $\qquad\qquad y + 8\lambda(-2y\lambda) = 0$

3 $\quad x + \lambda 2y = 0 \to x = -2y\lambda$ $\qquad y(1 - 16\lambda^2) = 0$

$4x^2 + y^2 = 8$

$y = 0 \quad \vee \quad \lambda = \pm \frac{1}{4}$

$y = 0 \to 4x^2 = 8$ $\qquad x_1 = -2y \cdot \frac{1}{4} = -y/2 \qquad x_2 = 2y \cdot 1/4 = \frac{y}{2}$

1 $\qquad x = \pm\sqrt{2}$

1 $\quad [+\sqrt{2}, 0] \quad [-\sqrt{2}, 0]$ $\qquad 2y^2 = 8 \quad y = \pm 2$

f: $\quad 0 \qquad 0$ $\qquad$

$[1,2]$	$[-1,2]$	$[1,-2]$	$[-1,-2]$
2	−2	−2	2
MAX	MIN	MIN	MAX

③ $f_n = \dfrac{n^2 x}{1+n^4 x^2}$ $\quad x \in [0,1]$

(a) fix $x \in [0,1]$ $\quad x \neq 0$ $\quad \lim\limits_{n\to\infty} \dfrac{n}{n^4} \cdot \dfrac{x}{\frac{1}{n^4} + x^2} = 0 \cdot \dfrac{x}{0+x} = 0$

2 $\quad x = 0$ $\quad \lim\limits_{n\to\infty} \dfrac{0}{1+0} = 0$ $\qquad f = 0$ na $[0,1]$

(b) fix n: $\quad \tau_n = \sup \{ |f_n - f|, x \in [0,1] \}$

1+1 $\quad = \sup \{ \underbrace{\dfrac{n^2 x}{1+n^4 x^2}}_{g_n(x)} ; x \in [0,1] \}$

pro $x = 0$ je $g(0) = 0$

$x = 1$ $\quad g(1) = \dfrac{n^2}{1+n^4}$

1+2 $\quad g_n'(x) = \dfrac{n^2(1+n^4x^2) - n^2x \cdot 2x n^4}{(1+n^4x^2)^2}$

$n^2 + n^6 x^2 - 2x^2 n^6 = 0$

$x^2 (-n^6) = -n^2$

$x^2 = \dfrac{1}{n^4}$

$x = \pm \dfrac{1}{n^2}$ $\quad x > 0$

$x_0 = \dfrac{1}{n^2}$

1 $\quad$ pak $g\left(\frac{1}{n^2}\right) = \dfrac{n^2 \cdot \frac{1}{n^2}}{1+n^4 \cdot \frac{1}{n^4}} = \dfrac{1}{2}$

1 $\quad \lim\limits_{n\to\infty} \tau_n = \dfrac{1}{2} \neq 0$

1 $\quad \Rightarrow f_n \not\rightrightarrows$ na $[0,1]$