①

$$2e^x - \frac{1}{1-x} - 1 - x \qquad \boxed{4}$$

$$2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)\right)$$

$$-\left(1 + x + x^2 + x^3 + x^4 + o(x^4)\right) - 1 - x$$

$$\cancel{x^2} + \frac{1}{3}x^3 + \frac{1}{12}x^4 + 2\,o(x^4)$$

$$-\cancel{x^2} - x^3 - x^4 - o(x^4) \quad 2a \cdot 10m. \ (5)$$

$$-\frac{2}{3}x^3 - \frac{11}{12}x^4 + \underbrace{o(x^4) + o(x^4)}_{o(x^4)}$$

$$\arctan x - \sin x \qquad \boxed{5}$$

$$x - \frac{x^3}{3} + \frac{x^5}{5} + o(x^5)$$

$$-\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)\right) = \quad (5)$$

$$x^3\left(-\frac{1}{3} + \frac{1}{6}\right) + x^5\left(\frac{1}{5} - \frac{1}{5!}\right) + \underbrace{o(x^5) - o(x^5)}_{o(x^5)}$$

$$= -\frac{1}{6}x^3 + \frac{\overset{3}{\cancel{24}}}{120}x^5 + o(x^5) \qquad !$$

② $\cosh(2x)$ $\boxed{6}$

$2x \to 0$

$$1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!} + o\left((2x)^6\right)$$

$$1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6 + \underset{o(x^6)}{o(64x^6)}$$

$$g_1 = 64x^6 \quad g_2 = x^6 \qquad \lim_{x \to 0} \frac{g_1}{g_2} = 64 \in \mathbb{R}$$

$\arctan(-3x)$ $\boxed{5}$

$$-3x + \frac{(-3x)^3}{3!} + \frac{(-3x)^5}{5!} + o\left((-3x)^5\right)$$

$$= -3x + 9x^3 - \frac{243}{5}x^5 + o(x^5)$$

$$g_1 = 243x^5 \quad g_2 = x^5 \quad \lim_{x \to 0} \frac{g_1}{g_2} = 243 \in \mathbb{R}$$

③ $\sqrt{1+x^3} = (1+x^3)^{1/2}$ $\boxed{6}$

$r = 1/2$ $\qquad x^3 \to 0$

$$1 + \frac{1}{2}x^3 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(x^3)^2 + o\left((x^3)^2\right)$$

$$= 1 + \frac{1}{2}x^3 + -\frac{1}{8}x^6 + o(x^6)$$

$\ln(1-x^2)$ $\boxed{8}$

$$-x^2 - \frac{(-x^2)^2}{2} + \frac{(-x^2)^3}{3} - \frac{(-x^2)^4}{4} + o\left((-x^2)^4\right)$$

$$= -x^2 - \frac{x^4}{2} - \frac{x^6}{3} - \frac{x^8}{4} + o(x^8)$$

④ $x^2 \arccos x$ $\boxed{3}$

$$x^2\left(\frac{\pi}{2} - x - \frac{1}{6}x^3 + o(x^3)\right) \qquad x^2 \to 0 \text{ na } \mathcal{P}(0)$$

$$= x^2 \cdot \frac{\pi}{2} - x^3 - \frac{1}{6}x^5 + \underbrace{x^2\,o(x^3)}_{o(x^5)}$$

$x\sqrt[3]{1+x}$ $\boxed{4}$

$$x\left(1 + \frac{1}{3}x + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}x^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3 + o(x^3)\right)$$

$$= x + \frac{x^2}{3} - \frac{x^3}{9} + \frac{5}{81}x^4 + \underbrace{x\,o(x^3)}_{o(x^4)}$$

(5) $\ln(1+x)\tan x$ $\boxed{4}$

L_5: $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4)$

$\tan x$: $x + \frac{1}{3}x^3 + o(x^4)$

$$\ln(1+x)\tan x = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4)\right)\cdot \left(x + \frac{1}{3}x^3 + o(x^4)\right)$$

$$= x^2 + \frac{1}{3}x^4 + x\,o(x^4) - \frac{x^3}{2} - \frac{x^2}{2}\left(\frac{1}{3}x^3 + o(x^4)\right) + \frac{x^4}{3} + \frac{x^3}{3}\left(\frac{1}{3}x^3 + o(x^4)\right) + \left(-\frac{x^4}{4} + o(x^4)\right)\cdot\left(x + \frac{x^3}{3} + o(x^4)\right)$$

$$= x^2 + x^4\left(\frac{1}{3} + \frac{1}{3}\right) - \frac{x^3}{2} + o(x^4)$$

$$= x^2 - \frac{x^3}{2} + \frac{2}{3}x^4 + o(x^4)$$

$x^5 = o(x^4)$ z def

$x\,o(x^5) = o(x^6)$ což je $o(x^4)$ věčička

$e^x \sin x$ $\boxed{3}$

$$\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)\right)\left(x - \frac{x^3}{6} + o(x^4)\right)$$

$$= x - \frac{x^3}{6} + o(x^4) + x^2 + x\left(-\frac{x^3}{6} + o(x^4)\right) + \frac{x^3}{2} + \frac{x^2}{2}\left(-\frac{x^3}{6} + o(x^4)\right) + \left(\frac{x^3}{6} + o(x^3)\right)\left(x - \frac{x^3}{6} + o(x^4)\right)$$

$$= x + x^2(1) + x^3\left(-\frac{1}{6} + \frac{1}{2}\right) + o(x^4)$$

$$= x + x^2 + \frac{x^3}{3} + o(x^4)$$

(6) $\cos(\sin x)$ $\boxed{4}$

$\sin x = x - \frac{x^3}{6} + o(x^4)$

$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{24} + o(y^5)$

$$1 - \frac{1}{2}\left(x - \frac{x^3}{6} + o(x^4)\right)^2 + \frac{1}{24}\left(x - \frac{x^3}{6} + o(x^4)\right)^4 + o\left(\left(x - \frac{x^3}{6} + o(x^4)\right)^5\right)$$

$$1 - \frac{1}{2}\left(x^2 - \frac{x^4}{3} + \frac{x^6}{36} + 2x\,o(x^4) - \frac{x^3}{3}o(x^4) + o(x^4)o(x^4)\right) + \frac{1}{24}x^4 + o(x^4)\ldots + o(x^5)$$

$$= 1 - \frac{x^2}{2} + x^4\left(\frac{1}{6} + \frac{1}{24}\right) + o(x^5) = 1 - \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^4)$$

zdův.: $o(x^4)o(x^4) = o(x^8) \to o(x^4)$

$$\lim_{x\to 0} \frac{(x - x^3/6 + o(x^4))^5}{x^5} = 1 + 0 + \cdots \to o(x^5)$$

$\ln(\cos x)$ $\boxed{6}$

$$\cos x = 1 \underbrace{- \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{6!} + o(x^7)}_{y}$$

$$\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$$

$$\ln(\cos x) = -\frac{x^2}{2} + \frac{1}{24}x^4 - \frac{x^6}{6!} + o(x^7) - \frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{6!} + o(x^7)\right)^2 + \frac{1}{3}\left(\qquad\right)^3 + o\left(\left(-\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{6!} + o(x^7)\right)^3\right)$$

$$= -\frac{x^2}{2} + \frac{1}{24}x^4 - \frac{x^6}{6!} - \frac{1}{2}\left(\frac{x^4}{4} + 2\cdot\frac{-x^6}{2\cdot 24}\right) + \frac{1}{3}\,\frac{-x^6}{8} + o(x^6)$$

$$= -\frac{x^2}{2} + x^4\left(-\frac{1}{8} + \frac{1}{24}\right) + x^6\left(-\frac{1}{6!} + \frac{1}{48} - \frac{1}{24}\right) + o(x^6)$$

$$= -\frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{1}{45}x^6 + o(x^6)$$