$$\lim_{x\to\infty} x^3\left(\tan\frac{1}{x} - \arctan\frac{1}{x}\right) = \frac{2}{3}$$

$$y = \frac{1}{x} \quad \to \quad y \to 0+$$

$$\lim_{y\to 0} \frac{\tan y - \arctan y}{y^3}$$

$$\arctan y = y - \frac{y^3}{3} + o(y^3)$$

$$\tan y = \frac{\sin y}{\cos y} = \frac{y - \frac{y^3}{3!} + o(y^3)}{1 - \frac{y^2}{2} + o(y^3)} = \frac{y - \frac{y^3}{6} + o(y^3)}{1 - \left(\frac{y^2}{2} + o(y^3)\right)} =$$

geom Σ

$$= \left(y - \frac{y^3}{6} + o(y^3)\right)\left(1 + \left(\frac{y^2}{2} + o(y^3)\right) + \left(\frac{y^2}{2} + o(y^3)\right)^2 + o\left(\left(\frac{y^2}{2} + o(y^3)\right)^2\right)\right)$$

$$= y + \frac{y^3}{2} + o(y^3) - \frac{y^3}{6} = y + \frac{1}{3}y^3 + o(y^3)$$

tedy $\tan y - \arctan y = \frac{2}{3}y^3 + o(y^3)$

$$\lim_{y\to 0} \frac{\frac{2}{3}y^3 + o(y^3)}{y^3} = \frac{2}{3} + 0$$

VOLSF $\quad f(y) = \dfrac{\tan y - \arctan y}{y^3} \qquad \lim_{y\to 0} f(y) = \frac{2}{3}$

$\quad g(x) = 1/x \qquad \lim_{x\to\infty} \frac{1}{x} = 0$

(P) $\frac{1}{x} \neq 0$ na $P(\infty, 1)$

<u>Závěr:</u> $\lim_{x\to\infty} x^3\left(\tan\frac{1}{x} - \arctan\frac{1}{x}\right) = \frac{2}{3}$