(5) Mathyz

1)(a) $\cosh x + \sinh x = e^x$

$\hookrightarrow \frac{1}{2}(e^x + e^{-x} + e^x - e^{-x}) = \frac{1}{2} \cdot 2 \cdot e^x = e^x$

(b) $\cosh^2 x - \sinh^2 x = 1$

$\hookrightarrow (\cosh x - \sinh x)(\cosh + \sinh x) =$

$= \frac{1}{2}(e^x + e^{-x} - e^x + e^{-x})(e^x) =$

$= \frac{1}{2} \cdot 2 \cdot e^{-x} \cdot e^x = 1$

(c) $\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = \frac{1}{2}(e^{2x} + e^{-2x})$

$\cosh^2 x + \sinh^2 x = \frac{1}{4}(e^x + e^{-x})^2 + \frac{1}{4}(e^x - e^{-x})^2$

$= \frac{1}{4}(e^{2x} + e^{-2x} + 2e^x e^{-x} + e^{2x} + e^{-2x} - 2e^x e^{-x})$

$= \frac{1}{4}(2e^{2x} + 2e^{-2x})$

(d) $\sinh 2x = 2 \sinh x \cosh x$

$\sinh 2x = \frac{1}{2}(e^{2x} - e^{-2x})$

$2 \sinh x \cosh x = 2 \cdot \frac{1}{2}(e^x - e^{-x})\frac{1}{2}(e^x + e^{-x})$

$= \frac{1}{2}(e^{2x} - e^{-2x})$

(e) $\cosh(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)}) = \frac{1}{2}(e^x + e^{-x}) = \cosh x$

(f) $\sinh(-x) = \frac{1}{2}(e^{-x} - e^{-(-x)}) = \frac{1}{2}(e^{-x} - e^x) =$

$= -\frac{1}{2}(e^x - e^{-x}) = -\sinh x$

g) $\coth(-x) = \dfrac{\cosh(-x)}{\sinh(-x)} = \dfrac{\cosh(+x)}{-\sinh x} = \coth x$

h) $\tanh(-x) = \dfrac{\sinh(-x)}{\cosh(-x)} = -\dfrac{\sinh x}{\cosh x} = -\tanh x$

i) $\dfrac{1}{\cos^2 x} = 1 + \tan^2 x$

$$\shortparallel$$

$$1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

j) $\dfrac{1}{\cosh^2 x} = 1 - \tanh^2 x$

$$\shortparallel$$

$$1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

2a) $\sinh(\ln 3) = \dfrac{e^{\ln 3} - e^{-\ln 3}}{2} = \dfrac{3 - \frac{1}{3}}{2} = \underline{\underline{\dfrac{4}{3}}}$

b) $\cosh(\ln 2) = \dfrac{e^{\ln 2} + e^{-\ln 2}}{2} = \dfrac{2 + \frac{1}{2}}{2} = \underline{\underline{\dfrac{5}{4}}}$

c) $\coth(\ln \frac{1}{3}) = \dfrac{e^{\ln \frac{1}{3}} + e^{-\ln \frac{1}{3}}}{e^{\ln \frac{1}{3}} - e^{-\ln \frac{1}{3}}} = \dfrac{\frac{1}{3} + 3}{\frac{1}{3} - 3} = -\dfrac{5}{4}$

3a) $\sinh x = \frac{3}{4}$

protože $\cosh^2 x = 1 + \sinh^2 x$ a $\cosh x \geq 0$,

máme $\cosh x = 5/4$.

Víme, že $\sinh x + \cosh x = e^x$

$e^x = \frac{3}{4} + \frac{5}{4} = 2 \quad \rightarrow \quad \boxed{x = \ln 2}$

3 (b) $\cosh x = \frac{13}{5}$

$\frac{1}{2}(e^x + e^{-x}) = \frac{13}{5}$

$y + \frac{1}{y} = \frac{26}{5} \qquad e^x =: y$

$y^2 + 1 = \frac{26}{5}y$

$y^2 - \frac{26}{5}y + 1 = 0$

$y_{1,2} = \frac{\frac{26}{5} \pm \sqrt{\frac{26^2}{25} - 4}}{2}$

$= \frac{\frac{26}{5} \pm \sqrt{\frac{676}{25} - \frac{100}{25}}}{2}$

$= \frac{\frac{26}{5} \pm \frac{24}{5}}{2} = \begin{cases} 5 \\ \frac{1}{5} \end{cases}$

$y_1 = 5 \qquad y_2 = \frac{1}{5}$

$e^x = 5 \qquad e^x = \frac{1}{5}$

$\underline{x = \ln 5} \qquad \underline{y = -\ln 5}$

3d) $4 \cosh x + \sinh x = 4$

$4(e^x + e^{-x}) + e^x - e^{-x} = 4 \cdot 2 \qquad e^x = y$

$5e^x + 3e^{-x} = 8$

$5y + \frac{3}{y} = 8$

$5y^2 - 8y + 3 = 0$

$y_{1,2} = \frac{8 \pm \sqrt{64 - 60}}{10}$

$y_{1,2} = \frac{8 \pm 2}{10} = \begin{cases} 1 \\ \frac{3}{5} \end{cases}$

$e^x = 1 \qquad e^x = \frac{3}{5}$

$x = \ln 1 \qquad x = \ln \frac{3}{5}$

$\underline{x = 0} \qquad \underline{x = \ln 3 - \ln 5}$

Activity 6

Find the values of x for which

$$\cosh x = \frac{13}{5}$$

expressing your answers as natural logarithms.

Example

Solve the equation

3e) $$2\cosh 2x + 10\sinh 2x = 5$$

giving your answer in terms of a natural logarithm.

Solution

$$\cosh 2x = \frac{1}{2}\left(e^{2x} + e^{-2x}\right); \quad \sinh 2x = \frac{1}{2}\left(e^{2x} - e^{-2x}\right)$$

So $$e^{2x} + e^{-2x} + 5e^{2x} - 5e^{-2x} = 5$$

$$6e^{2x} - 5 - 4e^{-2x} = 0$$

$$6e^{4x} - 5e^{2x} - 4 = 0$$

$$\left(3e^{2x} - 4\right)\left(2e^{2x} + 1\right) = 0$$

$$e^{2x} = \frac{4}{3} \text{ or } e^{2x} = -\frac{1}{2}$$

The only real solution occurs when $e^{2x} > 0$

So $$2x = \ln\frac{4}{3} \Rightarrow x = \frac{1}{2}\ln\frac{4}{3}$$

Exercise 2B

1. Given that $\sinh x = \frac{5}{12}$, find the values of

   (a) $\cosh x$ (b) $\tanh x$ (c) $\operatorname{sech} x$

   (d) $\coth x$ (e) $\sinh 2x$ (f) $\cosh 2x$

   Determine the value of x as a natural logarithm.

2. Given that $\cosh x = \frac{5}{4}$, determine the values of

   (a) $\sinh x$ (b) $\cosh 2x$ (c) $\sinh 2x$

   Use the formula for $\cosh(2x + x)$ to determine the value of $\cosh 3x$.

$\cosh^2 x - \sinh^2 x = 1$

$\cosh x = \frac{13}{12}$

$\cosh^2 x - \frac{25}{144}$

$\cosh^2 x = \frac{144+25}{144}$

$\cosh^2 x = \frac{169}{144}$

(4) $\sinh x = \frac{5}{12}$

(a) $\cosh x$ $\quad \cosh x = \sqrt{1 + \sinh^2 x}$

$$\cosh x = \sqrt{1 + \frac{25}{144}}$$

$$\cosh x = \sqrt{\frac{169}{144}}$$

$$\cosh x = \frac{13}{12}$$

// $\cosh x \geq 0$ $\forall x$
lahko v živo odkorenit

(b) $\coth x = \frac{\cosh x}{\sinh x} = \frac{\frac{13}{12}}{\frac{5}{12}} = \frac{13}{5}$

(c) $\tanh x = \frac{5}{13}$

(d) $\sinh 2x = 2 \sinh x \cosh x = 2 \cdot \frac{5}{12} \cdot \frac{13}{12} = \frac{65}{72}$

(e) $\cosh 2x = \cosh^2 x + \sinh^2 x = \frac{13^2}{12^2} + \frac{5^2}{12^2} = \frac{194}{144} = \frac{97}{72}$

5) $\operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

položimo $\quad y = \frac{1}{2}(e^x - e^{-x})$ $\qquad e^x =: z$

$$2y = z - \frac{1}{z}$$

$$2yz = z^2 - 1$$

$$0 = z^2 - 2yz - 1$$

$$z_{1,2} = \frac{2y \pm \sqrt{4y^2+4}}{2}$$

$$z_{1,2} = y \pm \sqrt{y^2+1}$$

$z_2 = y - \sqrt{y^2+1} < 0$

$\rightsquigarrow$ zavrnimo

pol $\quad x = \ln\left(y + \sqrt{y^2+1}\right)$

6) a) $\operatorname{arg\,sinh} \frac{3}{4} = \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right) =$

$\ln\left(\frac{3}{4} + \sqrt{\frac{25}{16}}\right) = \underline{\underline{\ln 2}}$

b) $\operatorname{arg\,cosh} 2 = \ln\left(2 + \sqrt{2^2 - 1}\right) =$

$= \ln \underline{\underline{(2+\sqrt{3})}}$

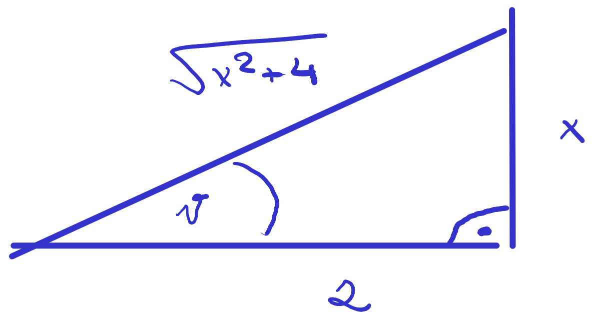
c) $\operatorname{arg\,tanh} \frac{1}{2} = \frac{1}{2} \ln\left(\frac{1+1/2}{1-1/2}\right) = \frac{1}{2} \underline{\underline{\ln 3}}$

7 a) $\sqrt{9-x^2} = \sqrt{9-9\sin^2\vartheta} = \sqrt{9(1-\sin^2\vartheta)} = 3\sqrt{\cos^2\vartheta}$

$= 3\cos\vartheta$ (protože $\vartheta \in (-\pi/2, \pi/2)$, je $\sqrt{\cos^2\vartheta} = |\cos\vartheta| = \cos\vartheta$)

$$\cot\vartheta = \frac{\cos\vartheta}{\sin\vartheta} = \frac{\frac{1}{3}\sqrt{9-x^2}}{\frac{1}{3}x} = \frac{\sqrt{9-x^2}}{x}$$

b) $x = 2\tan\vartheta$



$$\frac{1}{\cos^2\vartheta} = 1 + \tan^2\vartheta$$

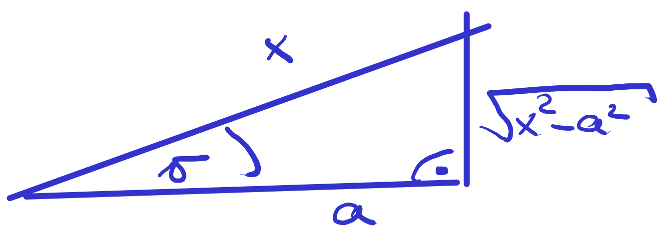
$$\frac{1}{\cos^2\vartheta} = 1 + \frac{x^2}{4}$$

$$\frac{1}{\cos^2\vartheta} = \frac{4+x^2}{4} \xrightarrow{\sqrt{}} \frac{1}{\cos\vartheta} = \frac{\sqrt{x^2+4}}{2}$$

$$\frac{2}{\cos\vartheta} = \sqrt{x^2+4}$$

$\|$ $\cos\vartheta > 0$ na $(-\pi/2, \pi/2)$ lze odmocnit

c) $x = \dfrac{a}{\cos\vartheta}$



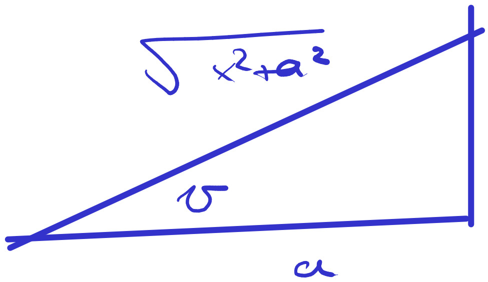
$$\frac{1}{\cos\vartheta} = \frac{x}{a}$$

$$\tan^2\vartheta = \frac{1}{\cos^2\vartheta} - 1 = \frac{x^2}{a^2} - 1 = \frac{x^2-a^2}{a^2}$$

$$\tan\vartheta = \frac{\sqrt{x^2-a^2}}{a}$$

dohromady $\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}$

d) $x = a \tan \vartheta$



$\sin \vartheta = ?$

$$\tan^2 \vartheta = \frac{\sin^2 \vartheta}{\cos^2 \vartheta} = \frac{\sin^2 \vartheta}{\frac{1}{\tan^2 \vartheta + 1}}$$

$$\rightarrow \quad \sin^2 \vartheta = \frac{\tan^2 \vartheta}{1 + \tan^2 \vartheta}$$

$$\sin \vartheta = \frac{\tan \vartheta}{\sqrt{1 + \tan^2 \vartheta}} = \frac{\frac{x}{a}}{\sqrt{1 + \frac{x^2}{a^2}}} = \frac{x}{\sqrt{a^2 + x^2}}$$

při odmocňování pozor na znaménka

⑧ $x = 2\cosh v$

$4\cosh v \sinh v = ?$

$$\sinh^2 v = \cosh^2 v - 1$$
$$= \frac{x^2}{4} - 1 = \frac{x^2-4}{4}$$

$$|\sinh v| = \frac{1}{2}\sqrt{x^2-4}$$

pak $4\cosh v \sinh v = \mathrm{sgn}(v) \cdot 4 \cdot \frac{x}{2} \cdot \frac{1}{2}\sqrt{x^2-4}$

znaménko v