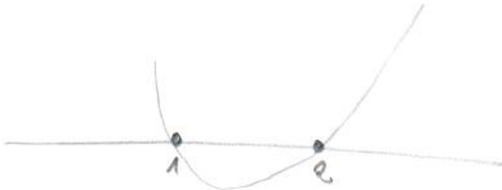
(1a) $(x-2)(x+3) \geq 4x-8$

$x^2+3x-2x-6 \geq 4x-8$

$x^2-3x+2 \geq 0$

$(x-2)(x-1) \geq 0$

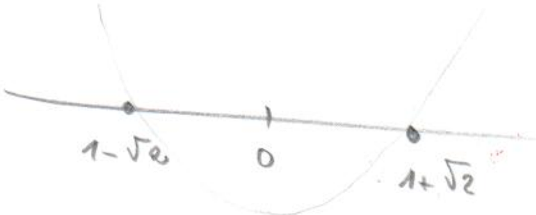



$x \in (-\infty, 1] \cup [2, \infty)$

(1b) $\dfrac{2x^2+1}{x^2+2x+2} < 1$

$\dfrac{2x^2+1-(x^2+2x+2)}{x^2+2x+2} < 0$

$\dfrac{x^2-2x-1}{x^2+2x+2} < 0$




$x^2+2x+2 > 0 \quad \forall x \in \mathbb{R}$

↖ ostrá nerovnost

$x^2-2x-1=0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4+4}}{2}$

$x_{1,2} = \dfrac{2 \pm \sqrt{8}}{2}$

$x_{1,2} = 1 \pm \sqrt{2}$

$x \in (1-\sqrt{2}, 1+\sqrt{2})$

(1c) $\dfrac{x-8}{x-9} \geq x$

$x \neq 9$

$\dfrac{x-8-x(x-9)}{x-9} \geq 0$

$\dfrac{-x^2+10x-8}{x-9} \geq 0$

| | $5-\sqrt{17}$ | | $9$ | | $5+\sqrt{17}$ | |
|---|---|---|---|---|---|---|
| $x-9$ | $-$ | | $-$ | $\times$ | $+$ | | $+$ |
| $-x^2+10x-8$ | $-$ | $\times$ | $+$ | | $+$ | $\times$ | $-$ |
| | $+$ | | $-$ | | $+$ | | $-$ |

$x \in (-\infty, 5-\sqrt{17}] \cup (9, 5+\sqrt{17}]$

$-x^2+10x-8=0$

$x_{1,2} = \dfrac{-10 \pm \sqrt{100-32}}{-2} \to x_{1,2} = \dfrac{-10 \pm \sqrt{4 \cdot 17}}{-2}$

$x_{1,2} = 5 \pm \sqrt{17}$

(1d) $\dfrac{x+5}{x+3} > \dfrac{x+4}{x+1}$ $\qquad x \neq -3 \qquad x \neq -1$

$$\frac{x+5}{x+3} - \frac{x+4}{x+1} > 0$$

$$\frac{(x+5)(x+1) - (x+4)(x+3)}{(x+3)(x+1)} > 0$$

$$\frac{x^2+6x+5 - x^2 - 7x - 12}{(x+3)(x+1)} > 0$$

$$\frac{-x-7}{(x+3)(x+1)} > 0$$

| | $-7$ | | $-3$ | | $-1$ | |
|---|---|---|---|---|---|---|
| $-x-7$ | + | | − | | − | − |
| $x+3$ | − | | − | | + | + |
| $x+1$ | − | | − | | − | + |
| | + | | − | | + | − |

$x \in (-\infty, -7) \cup (-3, -1)$

(1e) $\dfrac{x+6}{x-3} > \dfrac{x+4}{x+1}$ $\qquad x \neq 3 \qquad x \neq -1$

$$\frac{x+6}{x-3} - \frac{x+4}{x+1}$$

$$\frac{(x+6)(x+1) - (x+4)(x-3)}{(x-3)(x+1)} > 0$$

$$\frac{x^2+7x+6 - x^2 - x + 12}{(x-3)(x+1)} > 0$$

$$\frac{6x+18}{(x-3)(x+1)} > 0$$

| | | $-3$ | | $-1$ | | $3$ | |
|---|---|---|---|---|---|---|---|
| $6x+18$ | − | | + | | + | | + |
| $x-3$ | − | | − | | − | | + |
| $x+1$ | − | | − | | + | | + |
| | − | | + | | − | | + |

$x \in (-3, -1) \cup (3, \infty)$

(2a) $|x-1| + |x-3| + |x-5| = 4$


(a) $x \in (-\infty, -1)$

$$-x+1 \quad -x+3 \quad -x+5 = 4$$
$$-3x = -5$$
$$x = \frac{5}{3}$$

$5/3 \notin (-\infty, -1)$

(b) $x \in [1, 3)$

$$x-1 \quad -x+3 \quad -x+5 = 4$$
$$-x = -3$$
$$x = 3$$

$3 \notin [1, 3)$

(c) $x \in [3, 5)$

$$x-1 + x-3 - x+5 = 4$$
$$x = 3$$

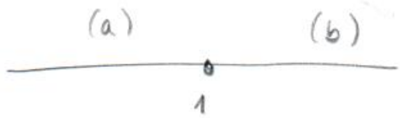
$x \in [5, \infty)$

(d) $x-1 + x-3 + x-5 = 4$
$$3x = 13$$
$$x = 13/3$$

$13/3 \notin [5, \infty)$

Závěr $\quad x = 3$

(2b) $||x-1| - 2| < 1$




(a) $x \in (-\infty, 1)$

$$|-x+1-2| < 1$$
$$|-x-1| < 1$$
$$(-1)(x+1)$$
$$|x+1| < 1$$


$x \in (-2, 0) \quad (\cap (-\infty, 1))$

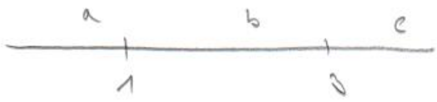
(b) $x \in [1, \infty)$

$$|x-1-2| < 1$$
$$|x-3| < 1$$


$x \in (2, 4) \quad (\cap [1, \infty))$

Závěr: $\quad x \in (-2, 0) \cup (2, 4)$

(2c) $|x-1| - |x-3| > x$




(a) $x \in (-\infty, 1)$

$$-x-1-(-x+3) > x$$
$$-x-1+x-3 > x$$
$$-4 > x$$

$x \in (-\infty, -4)$

(b) $x \in [1, 3)$

$$x-1-(-x+3) > x$$
$$x-1+x-3 > x$$
$$x > 4$$

nelze

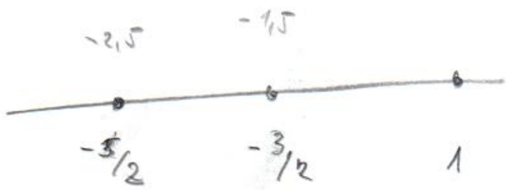
(c) $x \in [3, \infty)$

$$x-1-(x-3) > x$$
$$2 > x$$

nelze

závěr: $x \in (-\infty, -4)$

(2d) $|2x+3| + |2x+5| > |x-1|$




(a) $x \in (-\infty, -5/2)$

$$-2x-3 \; -2x-5 > -x+1$$
$$-3x > 9$$
$$-9 > 3x$$
$$-3 > x$$

$x \in (-\infty, -3)$

(b) $x \in [-5/2, -3/2)$

$$-2x-3 \; +2x+5 > -x+1$$
$$x > -1$$

nelze

(c) $x \in [-3/2, 1)$

$$2x+3+2x+5 > -x+1$$
$$5x > -7$$
$$x > -\frac{7}{5}$$

$x \in (-7/5, 1)$

(d) $x \in [1, \infty)$

$$2x+3+2x+5 > x-1$$
$$3x > -9$$
$$x > -3$$

$x \in [1, \infty)$

Závěr $x \in (-7/5, \infty) \cup (-\infty, -3)$

(2e) 1. test 3A

(1) $|x+2| > |x| - x$

| | $-2$ | | $0$ | |
|---|---|---|---|---|
| $x+2$ | $-$ | $+$ | | $+$ |
| $x$ | $-$ | $-$ | | $+$ |

(a) $x \in (-\infty, -2)$

$-x-2 > -x-x$

$x > 2$

nelze

(b) $x \in (-2, 0)$

$x+2 > -x-x$

$3x > -2$

$x > -\frac{2}{3}$

tedy $x \in (-\frac{2}{3}, 0)$

(c) $x \in (0, \infty)$

$x+2 > x-x$

$x > -2$

tedy $x \in (0, \infty)$

(d) hraniční body

$x = -2$

$0 > 2-(-2)$

$0 > 4$ ne

$x = 0$

$2 > 0$ ✓

celkem $x \in (-\frac{2}{3}, \infty)$

2f

# 1. test 3B

(1) $|x+2| > |x+1| + x$

| | | $-2$ | | $-1$ | |
|---|---|---|---|---|---|
| $x+2$ | $-$ | $\times$ | $+$ | | $+$ |
| $x+1$ | $-$ | | $-$ | $\times$ | $+$ |

(a) $x \in (-\infty, -2)$

$-x-2 > -x-1+x$

$\boxed{-1 > x}$

$x \in (-\infty, -2)$

(b) $x \in (-2, -1)$

$x+2 > -x-1+x$

$\boxed{x > -3}$

$x \in (-2, -1)$

(c) $x \in (-1, \infty)$

$x+2 > x+1+x$

$\boxed{1 > x}$

$x \in (-1, 1)$

(d) hraniční body

$x = -2$

$0 > |-2+1| - 2$

$0 > -1$ ✓

$x = -1$

$|-1+2| > -1$

$1 > -1$ ✓

Závěr: $\boxed{x \in (-\infty, 1)}$

2g

1. test 2A

(1) $|x-|x+2||<x$

(a) $x+2 \geq 0$

$\boxed{x \geq -2}$

$|x-x-2|<x$

$\boxed{2<x}$

(b) $x+2<0$

$\boxed{x<-2}$

$|x+x+2|<x$

$|2(x+1)|<x$

(b.1) $x+1 \geq 0$

$x \geq -1$

nelze

(b.2) $x+1<0$

$\boxed{x<-1}$

$-2x-2<x$

$-2<3x$

$\boxed{-\frac{2}{3}<x}$

nelze

Závěr $x \in (2, \infty)$

2a

# 1. Test 2B

(1) $|x+|x+2||<4x$

(a) $x+2\geq 0$

$\boxed{x\geq -2}$

(b) $x+2<0$

$\boxed{x<-2}$

$|x+x+2|<4x$

$|2(x+1)|<4x$

$|x-x-2|<4x$

$|-2|<4x$

$2<4x$

$\boxed{\frac{1}{2}<x}$

nelze

(a1) $x+1\geq 0$

$\boxed{x\geq -1}$

$2x+2<4x$

$2<2x$

$\boxed{1<x}$

(a2) $x+1<0$

$\boxed{x<-1}$

$-2x-2<4x$

$-2<6x$

$-\frac{1}{3}<x$

nelze

závěr $\boxed{x>1}$

(3a)

3B

(1) $a, x \in \mathbb{R}$

$$|x| + |x+7| < a$$

- $x \in (-\infty, -7)$

$-x - x - 7 < a$

$\boxed{-2x < a+7}$

$\boxed{-\frac{a+7}{2} < x}$

$x \in (-7, 0)$

$-x + x + 7 < a$

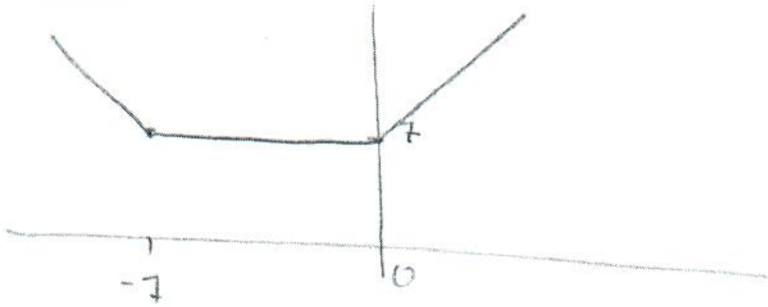
$\boxed{7 < a}$

$x \in (0, \infty)$

$x + x + 7 < a$

$\boxed{2x < a-7}$

$x \in \left(\frac{a-7}{2}\right)$




Řešení:

$a \in (-\infty, 7]$ $\emptyset$

$a \in (7, \infty)$ $x \in \left(-\frac{a+7}{2}, \frac{a-7}{2}\right)$

3b

2B

(1) $|x(x+2)| > a$


(a) $x(x+2) > 0$

$x^2 + 2x - a > 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4+4a}}{2}$

$a > -1$

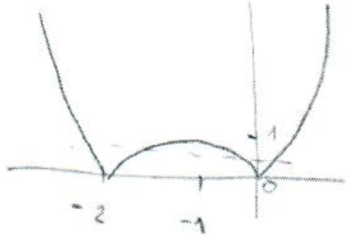
(b) $x(x+2) < 0$

$-x^2 - 2x - a > 0$

$x^2 + 2x + a < 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4-4a}}{2}$

$a < 1$




Pro

$a \in (-\infty, 0)$ $\quad x \in \mathbb{R}$

$a = 0$ $\quad x \in \mathbb{R} \setminus \{-2, 0\}$

$a \in (0, 1)$ $\quad x \in (-\infty, -1-\sqrt{1+a}) \cup (-1-\sqrt{1-a}, -1+\sqrt{1-a}) \cup (-1+\sqrt{1+a}, \infty)$

$a = 1$ $\quad x \in (-\infty, -1 - \frac{\sqrt{8}}{2}) \cup (-1 + \frac{\sqrt{8}}{2}, \infty)$

$a > 1$ $\quad x \in (-\infty, -1-\sqrt{1+a}) \cup (-1+\sqrt{1+a}, \infty)$

3c



(1) $x, a \in \mathbb{R}$

$||x| - 2| < a$

- $x = 0$

$\boxed{2 < a}$

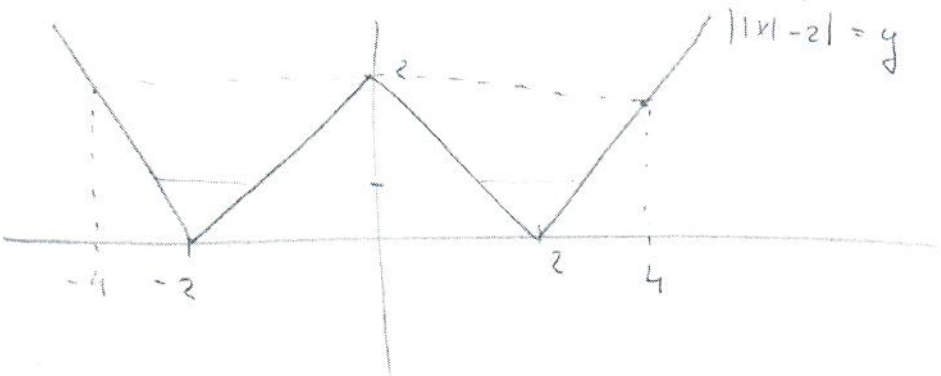
- $x > 0$

$|x - 2| < a$

- $x < 0$

$|-x - 2| < a$

- $x > 2$: $x - 2 < a$, $\boxed{x < a + 2}$
- $x < 2$: $-x + 2 < a$, $\boxed{2 - a < x}$
- $x = 2$: $\boxed{0 < a}$
- $x > -2$: $x + 2 < a$, $\boxed{x < a - 2}$
- $x < -2$: $-x - 2 < a$, $\boxed{-2 - a < x}$
- $x = -2$: $\boxed{0 < a}$




Řešení

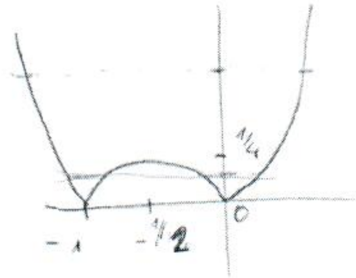
$a \in (-\infty, 0]$ $\quad \emptyset$

$a \in (0, 2]$ $\quad x \in (-2 - a, -2 + a) \cup (2 - a, 2 + a)$

$a \in (2, \infty)$ $\quad x \in (-2 - a, 2 + a)$

(3el) (2) $|x^2+x| > a$

- pro $a < 0$ $\quad x \in \mathbb{R}$




- $x^2+x=0$

  $x(x+1)=0 \qquad x_1=0 \qquad x_2=-1$

(a) $x^2+x \geq 0$

$x \in (-\infty, -1] \cup [0, \infty)$

$x^2+x > a$

$x^2+x-a > 0$

$x_{1,2} = \dfrac{-1 \pm \sqrt{1+4a}}{2}$

(b) $x^2+x < 0$

$x \in (-1, 0)$

$-x^2-x > a$

$0 > x^2+x+a$

$x_{1,2} = \dfrac{-1 \pm \sqrt{1-4a}}{2}$

len, ak $1-4a \geq 0$

$\frac{1}{4} \geq a$

(1) $a < 0$: $x \in \mathbb{R}$

(2) $a = 0$: $\quad x \in \mathbb{R} \setminus \{-1, 0\}$

(3) $a \in (0, \frac{1}{4})$ $\quad x \in \left(-\infty, -\frac{1}{2} - \frac{\sqrt{1+4a}}{2}\right) \cup \left(-\frac{1}{2} + \frac{\sqrt{1+4a}}{2}, \infty\right) \cup \left(\frac{-1-\sqrt{1-4a}}{2}, \frac{-1+\sqrt{1-4a}}{2}\right)$

(4) $a = \frac{1}{4}$ $\quad x \in \left(-\infty, -\frac{1}{2} - \frac{\sqrt{2}}{2}\right) \cup \left(\frac{-1+\sqrt{2}}{2}, \infty\right) \cancel{\cup \{-\frac{1}{2}\}}$

(5) $a > \frac{1}{4}$ $\quad x \in \left(-\infty, \frac{-1-\sqrt{1+4a}}{2}\right) \cup \left(\frac{-1+\sqrt{1+4a}}{2}, \infty\right)$

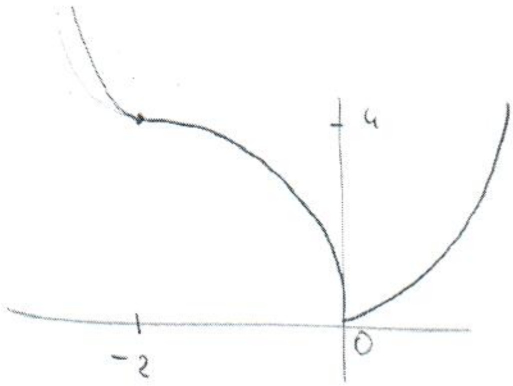
3e

(2) $|x^2+2x| < a+2x$

$x^2+2x=0$

$x(x+2)=0$

$|x^2+2x| - 2x < a$




(a) $x^2+2x > 0$

$x \in (-\infty,-2) \cup (0,\infty)$

$x^2+2x < a+2x$

$x^2 < a$

$x_1, x_2 = \pm\sqrt{a} \rightarrow a > 0$

(b) $x^2+2x<0$

$x \in (-2,0)$

$-x^2-2x < a+2x$

$0 < x^2+4x+a$

$x_{1,2} = \dfrac{-4 \pm \sqrt{16-4a}}{2}$

$x_{1,2} = -2 \pm \sqrt{4-a}$

$a<4$

(1) $a \leq 0$ $\emptyset$

(2) $a \in (0,4)$ $x \in [0,\sqrt{a}) \cup (-2+\sqrt{4-a}, 0)$

$= (-2+\sqrt{4-a}, \sqrt{a})$

(3) $a \in [4,\infty)$ $x \in (-\sqrt{a}, \sqrt{a})$

4

$$\frac{x+4}{x-3} \le 0 \qquad / \cdot (x-3)$$

co když $(x-3) < 0$? Pak se mění znaménko nerovnice

$x + 4 \le 0$

$x \le 4$

$x \in (-\infty, -4]$

5

(a) $(x+3)(x-2) \geq 0$ $\quad x^2+x-6 \geq 0$

(b) $(x+1)(x-5) < 0$ $\quad x^2-4x-5 < 0$

(c) $(x+6)^2 \leq 0$ $\quad x^2+12x+36 \leq 0$

(d) napr. $\quad x^2+42 < 0$

(6) $c \in \mathbb{R}$ $\qquad f(x) = \dfrac{x^2 + 2x + c}{x^2 + 4x + 3c}$

cíl: $\forall y \in \mathbb{R}$ najít $x \in \mathbb{R}$ tak, že

$$\frac{x^2+2x+c}{x^2+4x+3c} = y$$

(a) podmínky

$$x^2+4x+3c \neq 0 \qquad x_{1,2} = \frac{-4 \pm \sqrt{16-12c}}{2}$$

pro $c > \frac{4}{3}$ nenastane

(b) $x^2+2x+c = y(x^2+4x+3c)$

$$(1-y)x^2 + (2-4y)x + c(1-3y) = 0$$

aby rovnice měla řešení, potřebujeme $D \geq 0$

$$(2-4y)^2 - 4c(1-y)(1-3y) \geq 0$$

$$(4-3c)y^2 + (4c-4)y + (1-c) \geq 0$$

nerovnice musí platit pro $\forall y$, tedy její diskriminant musí být $\leq 0$

$$(4c-4)^2 - 4(4-3c)(1-c) \leq 0$$

$$c^2 - c \leq 0$$

$$c(1-c) \leq 0$$

$$\Rightarrow \boxed{0 \leq c \leq 1}$$

Závěr: $\underline{0 \leq c \leq 1}$