(1) $\lim\limits_{x\to 0} \dfrac{x\cos x - \arctan x}{x^3}$

$$\cos x = 1 - \frac{x^2}{2} + o(x^3) \qquad \arctan x \underset{x\to 0}{=} x - \frac{x^3}{3} + o(x^4) \qquad x\cos x = x - \frac{x^3}{2} + o(x^4)$$

$$\lim_{x\to 0} \frac{x - \frac{x^3}{2} + o(x^3) - x + \frac{x^3}{3} + o(x^3)}{x^3} = \lim \frac{x^3\left(-\frac{1}{2}+\frac{1}{3}\right)}{x^3} + \frac{o(x^3)}{x^3} = \underline{\underline{-\frac{1}{6}}} + 0$$

(2) $T_3^{f,0}$ $\qquad f(x) = \ln\left(\dfrac{1}{1-x}\right)$

$$\frac{1}{1-x} \underset{x\to 0}{=} 1 + \underbrace{x + x^2 + x^3 + o(x^3)}_{y} \qquad \ln(1+y) \underset{y\to 0}{=} y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$$

$$\ln\left(\frac{1}{1-x}\right) = x + x^2 + x^3 + o(x^3) - \frac{1}{2}\left(x + x^2 + x^3 + o(x^3)\right)^2 + \frac{1}{3}\left(x + x^2 + x^3 + o(x^3)\right)^3$$
$$+ o\left(\left(x + x^2 + x^3 + o(x^3)\right)^3\right)$$
$$= x + x^2 + x^3 - \frac{1}{2}\left(x^2 + 2x^3\right) + \frac{1}{3}x^3 + o(x^3)$$
$$= \underline{\underline{x + \frac{1}{2}x^2 + x^3\underbrace{\left(1 - 1 + \frac{1}{3}\right)}_{1/3}}} + o(x^3)$$

(3) $\displaystyle\sum \frac{n^4 - 2n^2 - 1}{\underbrace{n^5\sqrt{n} + \ln n}}$ $\qquad t = n^2 \qquad t^2 - 2t - 1 \qquad t_{1,2} = \dfrac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$a_n \geq 0$ pro $n \geq 2$

LSK s $b_n = \dfrac{n^4}{n^5\sqrt{n}} = \dfrac{1}{n\sqrt{n}} = \dfrac{1}{n^{3/2}}$ $\qquad \sum b_n$ K $\qquad b_n \geq 0$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim \frac{\frac{n^4 - 2n^2 - 1}{n^5\sqrt{n} + \ln n}}{\frac{n^4}{n^5\sqrt{n}}} = \lim \frac{1 - \frac{2}{n^2} - \frac{1}{n^4}}{1 + \underbrace{\frac{\ln n}{n^5\sqrt{n}}}_{\text{škála}}} \overset{AL}{=} \frac{1 - 0 - 0}{1 + 0} = 1 \in (0, \infty)$$

Protože $\sum b_n$ K, tak i $\underline{\underline{\sum a_n}}$ K.

(1), 2

(1) $\lim\limits_{x\to 0} \dfrac{\sin x - xe^x + x^2}{x^3}$

$\sin x = x - \frac{x^3}{6} + o(x^3)$ $\quad x\to 0 \quad$ $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$ $\quad xe^x = x + x^2 + \frac{x^3}{2} + o(x^3)$

$$\lim_{x\to 0} \frac{x - \frac{x^3}{6} - x - x^2 - \frac{x^3}{2} + x^2 + o(x^3)}{x^3} = \lim \frac{x^3(-\frac{1}{6} - \frac{1}{2})}{x^3} + \frac{o(x^3)}{x^3} = \underline{\underline{-\frac{2}{3} + 0}}$$

(2) $T_4^{f,0}$ $\quad f = \sqrt[3]{\cos x}$

$x \to 0 \qquad y \to 0$

$\cos x = 1 \underbrace{- \frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)}_{y}$ $\qquad (1+y)^{1/3} = 1 + \frac{1}{3}y + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}y^2 + o(y^2)$

$$\sqrt[3]{\cos x} = 1 + \frac{1}{3}\left(-\frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right) + \frac{-1}{9}\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)\right)^2 + o\left(\left(-\frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right)^2\right)$$

$$= 1 + -\frac{1}{6}x^2 + \frac{x^4}{3\cdot 24} - \frac{1}{9}\left(\frac{x^4}{4}\right) + o(x^4)$$

$$= 1 - \frac{1}{6}x^2 + x^4\left(\frac{1}{72} - \frac{1}{36}\right) + o(x^4) = \underline{\underline{1 - \frac{x^2}{6} - \frac{x^4}{72}}} + o(x^4)$$

(3) $\sum\limits_{n=1}^{\infty} \underbrace{\frac{n\, 2^{2n}}{n!}}_{a_n > 0}$

d'A

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{\frac{(n+1)\, 2^{2(n+1)}}{(n+1)!}}{\frac{n\cdot 2^{2n}}{n!}} = \lim \frac{n!}{(n+1)!} \cdot \frac{n+1}{n} \cdot \frac{2^{2n+2}}{2^{2n}} =$$

$$= \lim \frac{1}{n}\, 2^2 = 0\cdot 4 = 0 < 1$$

tedy $\sum a_n$ $\underline{\underline{K}}$

(1) $\quad$ (1) N

$$\lim_{x\to 0} \frac{\ln(1+x^2) - x\arcsin x}{x^4}$$

$$\ln(1+y) = y - \frac{y^2}{2} + o(y^2) \quad y\to 0 \qquad \ln(1+x^2) = x^2 - \frac{x^4}{2} + o(x^4) \quad x\to 0$$

$$\arcsin x = x + \frac{x^3}{6} + o(x^4) \qquad x\arcsin x = x^2 + \frac{x^4}{6} + o(x^4)$$

$$\lim_{x\to 0} \frac{x^2 - \frac{x^4}{2} + o(x^4) - x^2 - \frac{x^4}{6} + o(x^4)}{x^4} = \lim \frac{x^4\left(-\frac{1}{2}-\frac{1}{6}\right) + o(x^4)}{x^4} = \underline{\underline{-\frac{2}{3}}} + 0$$

(2) $T_3^{f,0}$ $\quad f = e^{\tan x}$

$$\tan x = \overbrace{x + \frac{x^3}{3} + o(x^3)}^{y} \quad x\to 0 \qquad e^y = 1 + y + \frac{y^2}{2} + \frac{y^3}{6} + o(y^3) \quad y\to 0$$

$$e^{\tan x} = 1 + x + \frac{x^3}{3} + o(x^3) + \frac{1}{2}\left(x + \frac{x^3}{3} + o(x^3)\right)^2 + \frac{1}{6}\left(x + \frac{x^3}{3} + o(x^3)\right)^3 + o\left(\left(x + \frac{x^3}{3} + o(x^3)\right)^3\right)$$

$$= 1 + x + \frac{x^3}{3} + \frac{1}{2}x^2 + \frac{1}{6}x^3 + o(x^3) =$$

$$= \underline{\underline{1 + x + \frac{x^2}{2} + \frac{x^3}{2}}} + o(x^3)$$

(3) $\sum (-1)^n \underbrace{\frac{n}{n^3-2}}_{b_n}$

$$\lim b_n = \lim \frac{1}{n^2 - 2/n} = 0$$

$$b_{n+1} \overset{?}{\leq} b_n$$

$$\frac{n+1}{(n+1)^3-2} \overset{?}{\leq} \frac{n}{n^3-2} \qquad \text{pro } n>1 \text{ je } n^3-2>0 \text{ a } (n+1)^3-2>0$$

$$(n+1)(n^3-2) \overset{?}{\leq} n\left(n^3+3n^2+3n+1-2\right)$$

$$\cancel{n^4} + n^3 - 2n - 2 \leq \cancel{n^4} + 3n^3 + 3n^2 - n$$

$$0 \leq 2n^3 + 3n^2 + n + 2 \checkmark$$

přes derivace: $\quad x \neq \sqrt[3]{2}$

$$f'(x) = \left(\frac{x}{x^3-2}\right)' = \frac{x^3-2 - x\cdot 3x^2}{(x^3-2)^2} = \underbrace{\frac{-2x^3-2}{(x^3-2)^2}}_{<0}$$

pro $x>1$ $\longrightarrow$

Tedy z Leibnize $\sum (-1)^n \frac{n}{n^3-2}$ k. $\qquad$ tedy $f \searrow$ tedy $b_n \searrow 0$