lok extr.

1e

$f(x,y) = 2x^3 + 9xy^2 + 15x^2 + 27y^2$

$\frac{\partial f}{\partial x} = 6x^2 + 9y^2 + 30x$

$\frac{\partial f}{\partial y} = 18xy + 54y$

$6x^2 + 9y^2 + 30x = 0$

$18xy + 54y = 0$

$\to y \cdot 18(x+3) = 0$

$y = 0 \quad \vee \quad x = -3$

$\to 6x^2 + 30x = 0$

$6x(x+5) = 0$

$x = 0 \quad x = -5$

$[0,0], \quad [-5,0]$

$54 + 9y^2 - 90 = 0$

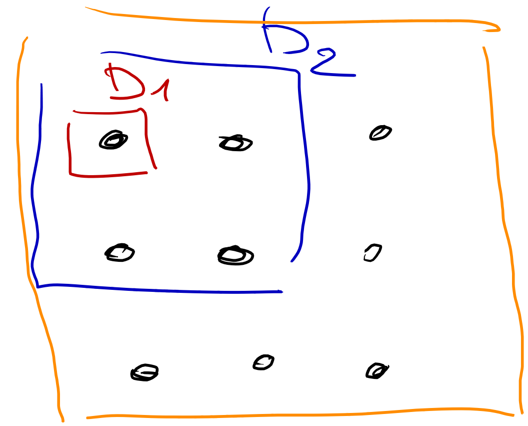
$9y^2 - 36 = 0$

$y^2 = 4$

$y = \pm 2$

$[-3, 2], \quad [-3, -2]$

$$\begin{pmatrix} \partial xx & \partial xy \\ 12x+30 & 18y \\ \partial yx & \partial yy \\ 18y & 18x+54 \end{pmatrix}$$



$[0,0]$ $\quad \begin{vmatrix} 30 & 0 \\ 0 & 54 \end{vmatrix}$ $\quad D_1 = 30 > 0$ pos.

$D_2 = 30 \cdot 54 > 0$ definitní

$\rightarrow$ lok. minim

$[-5,0]$ $\quad \begin{vmatrix} -30 & 0 \\ 0 & -36 \end{vmatrix}$ $\quad D_1 < 0$ neg. def.

$D_2 = +30 \cdot 36 > 0$

$\rightarrow$ lok. max

$[-3,2]$ $\quad \begin{vmatrix} -6 & 36 \\ 36 & 0 \end{vmatrix}$ $\quad D_1 = -6 < 0$ indef.

$D_2 = -36^2 < 0$ $\rightarrow$ není extrém

$[-3,-2]$ $\quad \begin{vmatrix} -6 & -36 \\ -36 & 0 \end{vmatrix}$ $\quad D_1 = -6 < 0$ indef.

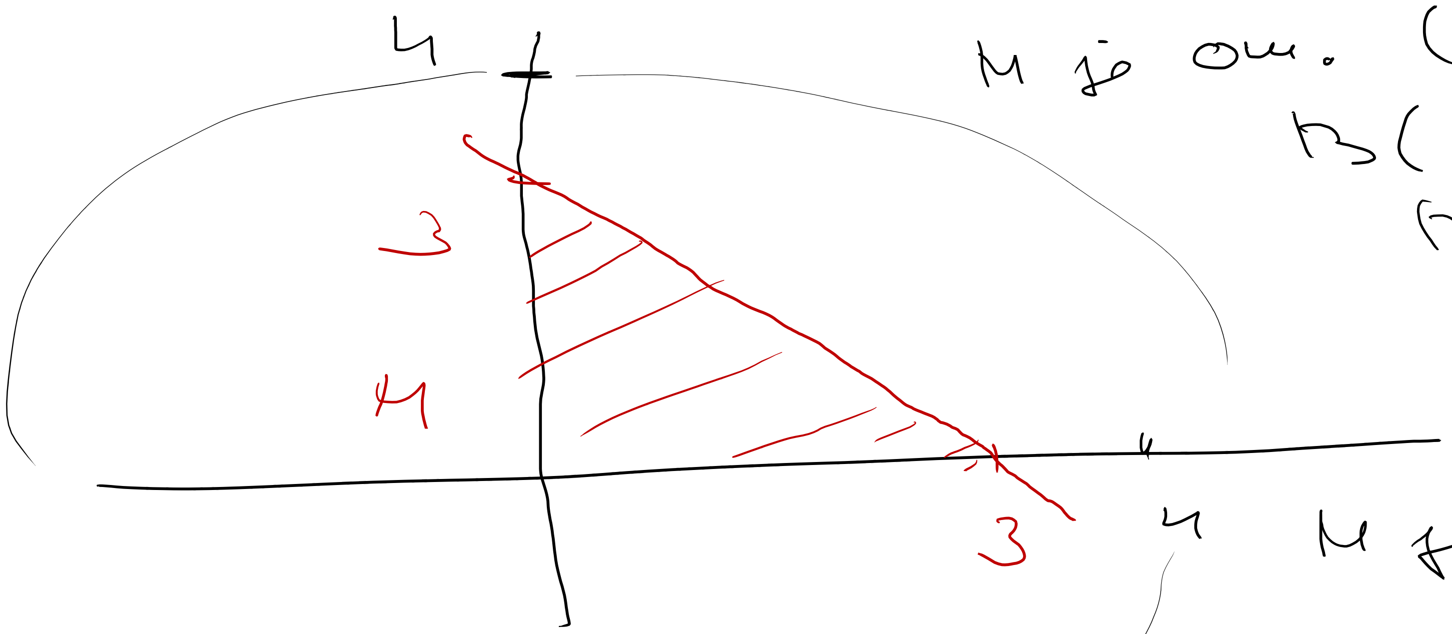
$D_2 = -36^2 < 0$ $\rightarrow$ není extr.

glob (4)

$f(x,y) = x^2 - 2y^2 + 4xy - 6x - 1$ $\qquad D_f = \mathbb{R}^2$

M: $\quad x \geq 0 \quad y \geq 0 \quad y \leq 3 - x$

M:



M je om. (vejde se B(0,4)) $B_4(0)$)

M je uz.

M uz: $\quad M_1 = \{[x,y] : x \geq 0\}$

$M =$ $\qquad g_1(x,y) = x \qquad M_1 = g_1^{-1}(\underbrace{[0,\infty)}_{I_1})$

$M_1 \cap M_2 \cap M_3$ $\qquad M_2 = \{ \qquad y \geq 0\}$

průnik uz $\qquad g_2(x,y) = y \qquad M_2 = g_2^{-1}(\underbrace{[0,\infty)}_{I_2})$

je $\boxed{\text{uz.}}$ $\qquad M_3 = \{ \quad : y \leq 3 - x\}$

$g_3 = y + x \qquad M_3 = g_3^{-1}(\underbrace{(-\infty,3]}_{I_3})$

g_1, g_2, g_3 spoj.
I_1, I_2, I_3 uz $\Big\} \longrightarrow M_1, M_2, M_3$ uz.

uz + om
$(v\ \mathbb{R}^4)$ $\longrightarrow$ $\boxed{\text{M kpt}}$

$f(x,y) = x^2 - 2y^2 + 4xy - 6x - 1$
f je spoj. (polynom) $\Big\}$ f na M nabývá extrému :)

• int M (M°)

$$\frac{\partial f}{\partial x} = 2x + 4y - 6$$

$$\frac{\partial f}{\partial y} = -4y + 4x$$

$$\begin{aligned} 2x + 4y - 6 &= 0 \\ -4y + 4x &= 0 \\ \hline 6x &= 6 \end{aligned}$$

$\boxed{y = 1} \leftarrow \boxed{x = 1}$

$A_1 = [1,1]$

•

$y = 0 \quad x \in (0,3)$ $\qquad$ $y = 3 - x \quad x \in (0,3)$

$x = 0 \quad y \in (0,3)$

$x^2 - 2y^2 + 4xy - 6x - 1$

$f(x,0) = x^2 - 6x - 1 =: h_1(x)$ $\qquad$ mimo

$h_1' = 2x - 6$ $\qquad$ $2x - 6 = 0$ $\qquad$ $\boxed{x = 3}$

$f(0,y) = -2y^2 - 1 =: h_2(y)$ $\qquad$ mimo interval

$h_2'(y) = -4y$ $\qquad$ $-4y = 0$ $\qquad$ $\boxed{y = 0}$

$f(x, 3-x) = x^2 - 2(3-x)^2 + 4x(3-x) - 6x - 1$

$= -5x^2 + 18x - 19 =: h_3(x)$

$h_3' = -10x + 18$ $\qquad$ $-10x = -18$

$y = 3 - \frac{9}{5} \longrightarrow \boxed{x = \frac{9}{5}}$ je na

$\boxed{y = \frac{6}{5}}$

• podezřelé body

$[1,1]$ $\qquad$ $[9/5, 6/5]$

• vrcholy $\quad [0,0] \quad [3,0] \quad [0,3]$

$f(1,1) = -4$

$f(0,0) = -1$ $\qquad$ f má glob. max $f(0,0) = -1$

$f(3,0) = -10$

$f(0,3) = -19$ $\qquad$ glob. min $f(0,3) = -19$

$f(9/5, 6/5) = -\frac{14}{5}$