

$$x^y = e^{y \log x} \quad \underbrace{x > 0, y \in \mathbb{R}}$$

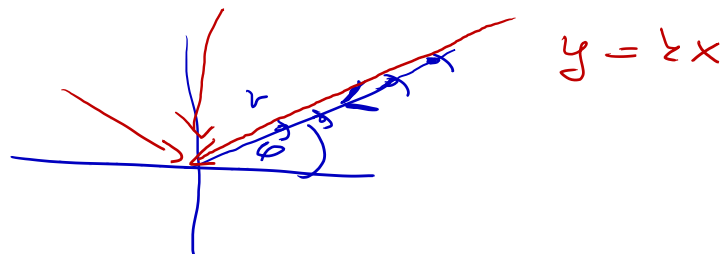
$g(r, \varphi)$

$$|f(r, \varphi)| \leq g(r) \rightarrow 0$$

$$\downarrow$$

$$0$$

$$\lim_{r \rightarrow 0} g(r, \varphi) = 0$$



$$f(x, y) = 4 - x^2 - y^2$$

$$\frac{\partial f}{\partial x} = -2x$$

$$\nabla f(x, y) = (-2x, -2y)$$

$$\frac{\partial f}{\partial y} = -2y$$

$$\nabla f(1, 0) = (-2, 0)$$

do vetor $\vec{v} = (2, 3)$

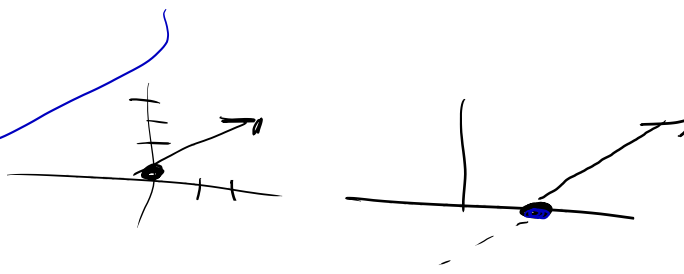
o vetor $(1, 0) = a$

$$D_{\vec{v}}(a) = (-2, 0) \cdot (2, 3) = -4 + 0 = -4$$

$$(-2, 0) \cdot (1, 15) = -2$$

T

$$(2, 3) \Rightarrow \frac{(2, 3)}{\sqrt{4+9}} = \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right)$$



$$\frac{\partial^2 f}{\partial x_j \partial x_i} = \frac{\partial}{\partial x_j} \left(\frac{\partial f}{\partial x_i} \right)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right)$$

$$f(x, y) = x^2 y^3$$

$$\frac{\partial f}{\partial y} = \underbrace{3y^2 x^2}_{\text{treat } x}$$

$$6xy^2 = f(x, y)$$

∴

$$\frac{\partial f}{\partial x} = 2xy^3$$

$$\frac{\partial^2 f}{\partial x \partial y} = \underbrace{6xy^2}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \underbrace{6xy^2}$$

$$\mathbb{R}^2 \xrightarrow{f} \mathbb{R}^3$$

$$\xrightarrow{g} \mathbb{R}^1$$

$$g(u, v, w) = uv - w$$

$$f(x, y) = (x^2y, x+y, y^3-2x)$$

$$f_1(x, y)$$

$$f_2(x, y)$$

$$f_3(x, y)$$

$$df = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} \end{pmatrix} = \begin{pmatrix} 2xy & x^2 \\ 1 & 1 \\ -2 & 3y^2 \end{pmatrix}$$

$$\text{v node } [1, 2]$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ -2 & 12 \end{pmatrix}$$

$$du \cdot g(f) \sim [1, 2]$$

$$f([1, 2]) = (2, 3, 6)$$

$$g(f([1, 2])) = g(2, 3, 6)$$

$$u \quad v \quad w$$

$$\text{v node } [2, 3, 6]:$$

$$g \begin{pmatrix} \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} & \frac{\partial g}{\partial w} \\ 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} 3 & 2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 & -1 \end{pmatrix}$$

$$\cdot \begin{pmatrix} \frac{\partial g(f)}{\partial x} & \frac{\partial g(f)}{\partial y} \\ 4 & 1 \\ 1 & 1 \\ -2 & 12 \end{pmatrix} = \begin{pmatrix} 16 & -7 \end{pmatrix}$$

$$\text{v node } [1, 2]$$