$$f(x,y) = x^2y - x^3 + 17y^2 - xy + 5$$

$$x^2 \cdot 4 - x^3 + 17 \cdot (4)^2 - x4 + 5$$

$$\frac{\partial f}{\partial x} = y \cdot 2x - 3x^2 + 0 - y \cdot 1 + 0$$

$x,\ y$ jako konst $\qquad [x,y] \in \mathbb{R}^2$

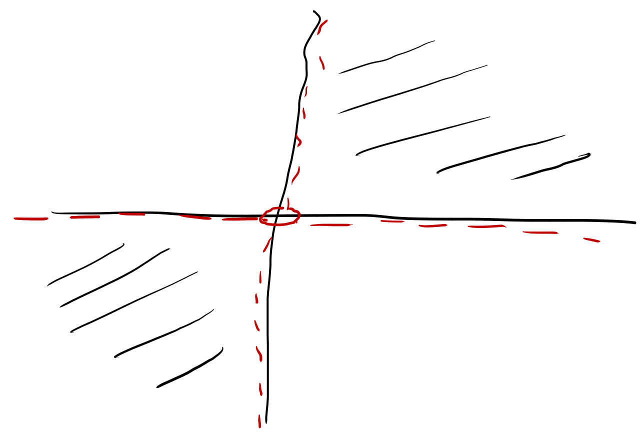
$$\frac{\partial f}{\partial y} = x^2 \cdot 1 - 0 + 34y - x \cdot 1 + 0$$

x jako konst.

$$f(x,y) = \frac{\ln(xy)}{xy} \qquad xy > 0$$

$$\frac{\partial f}{\partial x} = \frac{\frac{1}{xy} \cdot 1y \cdot xy - \ln(xy) \cdot 1y}{(xy)^2}$$

$$\frac{\partial f}{\partial y} = \frac{\frac{1}{xy} \cdot x \cdot xy - \ln(xy) \cdot 1 \cdot x}{(xy)^2}$$



$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f(x, y) \qquad a = [a_1, a_2]$$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(a_1 + h, a_2) - f(a_1, a_2)}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(a_1, a_2 + h) - f(a_1, a_2)}{h}$$

$|x| \quad \sqrt{x^2} \qquad (|x|)' = (\sqrt{x^2})' \Rightarrow$ případy $\Rightarrow \operatorname{sgn} x$

$x \neq 0$

2a) $f(x,y) = |x^2 - y^2|$

1) $D_f: [x,y] \in \mathbb{R}^2$

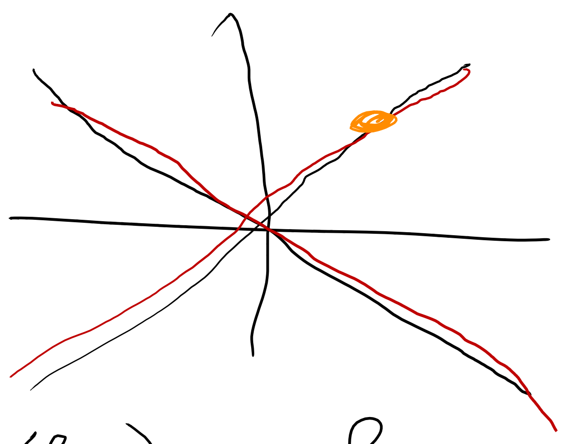
2) $\frac{\partial f}{\partial x} = \operatorname{sgn}(x^2-y^2) \cdot 2x$

$\frac{\partial f}{\partial y} = \operatorname{sgn}(x^2-y^2) \cdot (-2y)$

$D_{\frac{\partial f}{\partial x}}, D_{\frac{\partial f}{\partial y}}$: $x^2 - y^2 \neq 0 \qquad x^2 \neq y^2$

(3) co když $x^2 = y^2$?

fix $[x_0, y_0]: x_0^2 = y_0^2$



$\frac{\partial f}{\partial x}[x_0,y_0] = \lim_{h\to 0} \frac{f(x_0+h,y_0) - f(x_0,y_0)}{h}$

$= \lim_{h\to 0} \frac{|(x_0+h)^2 - y_0^2| - \overbrace{|x_0^2 - y_0^2|}^{0}}{h}$

$= \lim_{h\to 0} \frac{|x_0^2 + 2x_0h + h^2 - y_0^2|}{h}$

$= \lim_{h\to 0} \frac{|2x_0h + h^2|}{h}$

$= \lim_{h\to 0} \frac{|h|}{h} |2x_0 + h|$ $\quad -3+h$

$\lim_{h\to 0+} 1 \, |2x_0|$

$\lim_{h\to 0-} -1 \, |2x_0|$

$x_0 = 0 \qquad \lim_{h\to 0} \qquad = 0$

$x_0 \neq 0 \qquad \lim_{h\to 0} \qquad \nexists$

$\frac{\partial f}{\partial x}(x_0,y_0) = \begin{cases} \nexists \\ 0 \end{cases} \quad x_0 = 0 \; (y_0 = 0)$

$\frac{\partial f}{\partial y}$ vyjde stejně

$\frac{\partial f}{\partial y}(x_0,y_0) = \begin{cases} 0 & x_0 = 0 = y_0 \\ \nexists & \text{jinak} \end{cases}$

Pozn.

- $|\ | = 0$
- $\sqrt{\ } = 0$ $\qquad \sqrt{x^2}$
- $\arcsin(\ ) = \pm 1$
- f def po částech
- max, min

2c) $\sqrt{y + \sin x}$ $\qquad y \geq -\sin x$

dle $\qquad y > -\sin x$