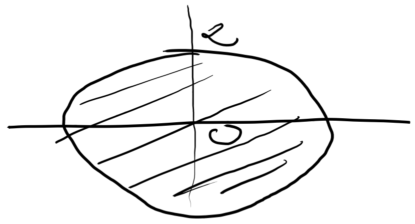
$$f(x,y) = x^2 + y^2 \qquad [x,y] \in \mathbb{R}^2$$

$$z = x^2 + y^2$$

$$f = \sqrt{4-(x^2+y^2)} \qquad 4-(x^2+y^2) \geq 0$$

$$4 \geq x^2 + y^2$$



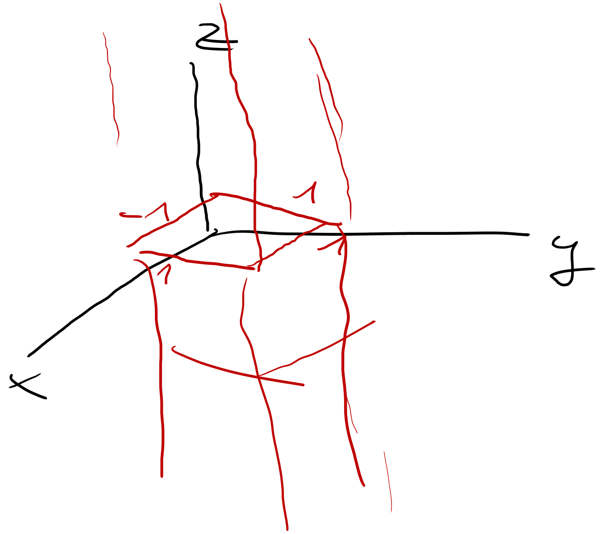
$$f(x,y,z) = \arcsin x + \arccos y + \arctan z$$

$D_f \subset \mathbb{R}^3$

$x \in [-1,1]$

$y \in [-1,1]$

$z \in \mathbb{R}$



$$\ln\left(\frac{x}{|x| - |y|}\right)$$

$x > 0$ & $|x| > |y|$

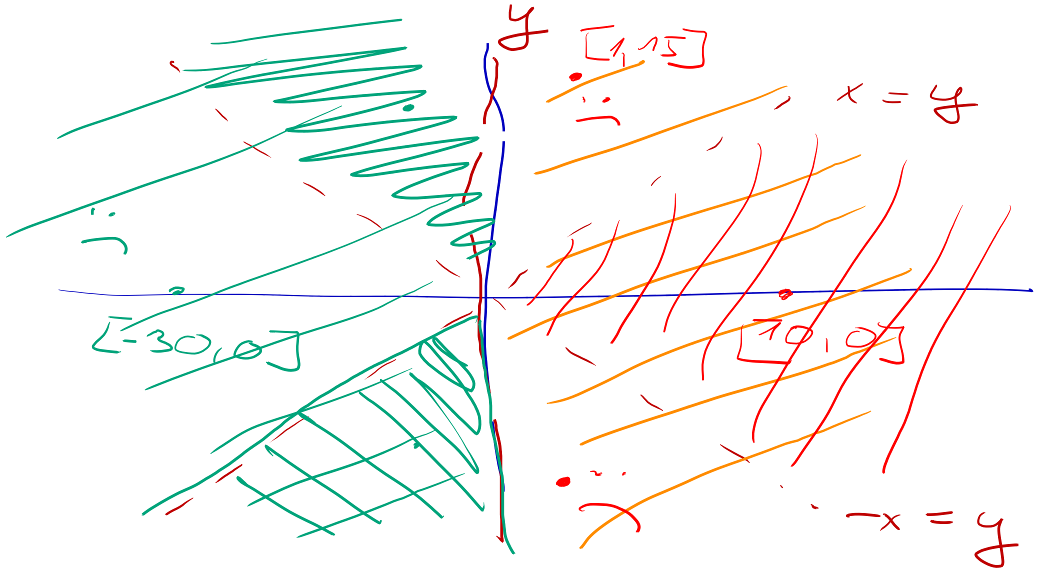
$\vee$ $x < 0$ & $|x| < |y|$

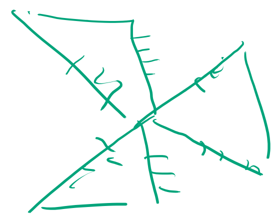
$x \neq 0$ $|x| \neq |y|$

$x = 0$

$x \neq y$

$x \neq -y$





$f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$X, \rho_2 \longrightarrow Y, \rho_2$

$$f = \frac{1}{x^2+y^2}$$

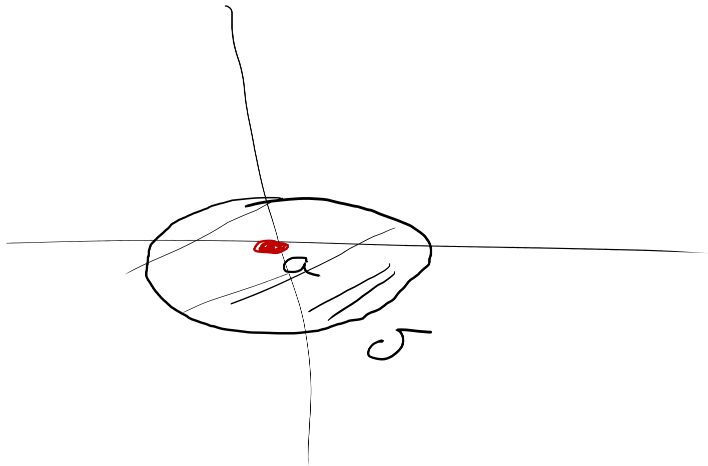
$$\lim_{[x,y]\to[0,0]} \frac{1}{x^2+y^2}$$

$\forall \varepsilon > 0 \;\; \exists \delta > 0$

$x \in B(a,\delta) \setminus \{a\}$

$\Rightarrow \;\; f(x) \in B(A,\varepsilon)$

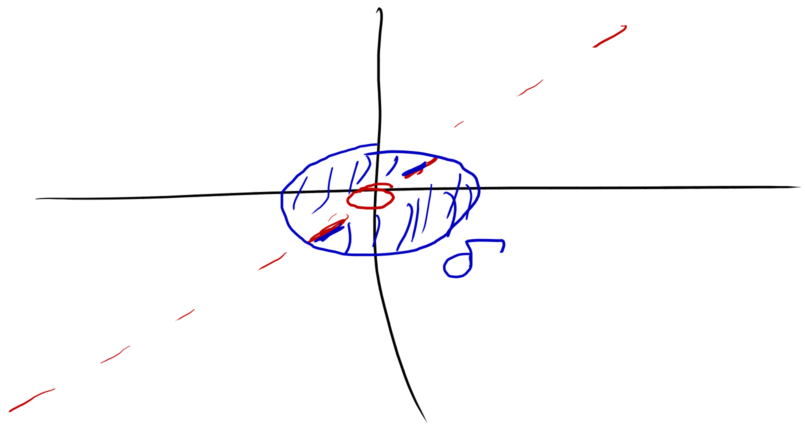
$D_f = \mathbb{R}^2 \setminus \{[0,0]\}$



$$f(x,y) = \frac{x-y}{x-y} \qquad D_f: \mathbb{R}^2 \setminus \{ x \neq y \}$$

$$\lim_{x,y \to [0,0]} \frac{x-y}{x-y} = 1$$

$[x,y] \in D_f$



$$\lim_{[x,y] \to [2,2]} \frac{x+3}{yx-2} = \frac{2+3}{2\cdot 2-2} = \frac{5}{2}$$

spoj.

$f(x,y) = x$ spojité

$f(x,y) = y$ —"—

$S \pm S$

$S \cdot S$

$\frac{S}{S}$ ne v 0

• $$\lim_{[x,y]\to[0,0]} \frac{x^2-y^2}{x-y} = \lim_{x,y\to 0,0} \frac{\cancel{(x-y)}(x+y)}{\cancel{x-y}} = 0+0=0$$

$$\frac{0}{0}$$

• $$\lim_{[x,y]\to[0,0]} \underbrace{(x+y)}_{\to 0} \cdot \underbrace{\sin\frac{1}{xy}}_{\text{om}} = 0$$

$$\lim_{x\to 0} \underbrace{x}_{\to 0}\cdot \underbrace{\sin\frac{1}{x}}_{\text{om.}}$$

• $$\lim_{x,y\to 0,0} \frac{1}{\sqrt{y^2+x^2}} \cdot \sin(\overbrace{x^2+y^2}^{z})$$

$$\lim_{z\to 0} \frac{\sin z}{z} = 1$$

$f(z)=\frac{\sin z}{z}$ $\quad \lim_{z\to 0} f = 1$

$g(x,y) = x^2+y^2$

$\lim_{x,y\to 0,0} g(x) = 0$

$$= \lim \frac{x^2+y^2}{\sqrt{x^2+y^2}} \cdot \frac{\sin(x^2+y^2)}{x^2+y^2}$$

$$= \lim \sqrt{x^2+y^2} \cdot \underbrace{\frac{\sin(x^2+y^2)}{x^2+y^2}}_{\to 1} = 0\cdot 1 = 0$$

$x^2+y^2 \neq 0$ na $P(0,\delta)$ ~~(S)~~ (P) ✓

3e

$$\lim_{x,y\to 0,0} x\frac{xy}{x^2+y^2}$$

$$\pm 2xy \le x^2+y^2$$

$$0 \le (x \mp y)^2$$

$$\left|x\frac{xy}{x^2+y^2}\right| \le \underbrace{\left|x\,\frac{1}{2}\,\cancel{\frac{x^2+y^2}{x^2+y^2}}\right|}_{0}$$

$$\lim \left|\frac{x^2y}{x^2+y^2}\right| = 0$$

$$\Rightarrow \lim \frac{x^2y}{x^2+y^2} = 0 \quad ✓$$

Nee x.

(1 v) $\lim_{x,y \to (0,0)} \frac{x-y^2}{x+y}$ $\nexists$

- $\lim_{x \to 0} \left(\lim_{y \to 0} \frac{x-y^2}{x+y} \right) = \lim_{x \to 0} \frac{x}{x} = 1$

$\lim_{y \to 0} \left(\lim_{x \to 0} \frac{x-y^2}{x+y} \right) = \lim_{y \to 0} \frac{-y^2}{y} = 0$

(4iv) $\lim\limits_{x,y\to 0,0} \dfrac{xy}{x^2+y^2}$

$$\lim_{x\to 0}\left(\lim_{y\to 0}\frac{xy}{x^2+y^2}\right)=\lim_{x\to 0}\frac{0}{x^2}=0$$

$$\lim_{y\to 0}\left(\lim_{x\to 0}\frac{xy}{x^2+y^2}\right)=\lim_{y\to 0}\frac{0}{y^2}=0$$

Nevíme nic

Zkusíme $\underline{y=x}$ $(y=kx)$ $y=0$

$$\lim_{x,y\to[0,0]}\frac{x\cdot x}{x^2+x^2}=\lim_{x\to[0,0]}\frac{1}{2}=\underline{\frac{1}{2}}$$

$\neq$

$$\lim_{x,y\to[0,0]}\frac{0}{x^2+0^2}=\underline{0}$$

$\to \dfrac{0}{x^2}$

$\to \lim \nexists$

5v)

$$\lim_{x,y\to[0,0]} \frac{x^2 y}{x^4+y^2} \quad \nexists$$

přímky $\quad y = kx \quad k \in \mathbb{R}\setminus\{0\}$

$$\lim_{x,y\to[0,0]} \frac{x^2\cdot kx}{x^4+k^2x^2} = \lim_{x,y\to[0,0]} \cancel{\frac{x^2}{x^2}}\cdot\frac{kx}{x^2+k^2}$$

$$= \frac{0}{0+k^2} = \underline{\underline{0}}$$

$$\neq$$

$$y = x^2$$

$$\lim_{x\to 0} \frac{x^2\cdot x^2}{x^4+(x^2)^2} = \lim_{x\to 0}\frac{x^4}{2x^4} = \underline{\underline{\frac{1}{2}}}$$

$$x = x_0^0 + \underbrace{r\cos\varphi}$$
$$y = y_0^0 + \underbrace{r\sin\varphi}$$

$r > 0 \quad \varphi \in [0, 2\pi]$

$$x^2 + y^2 = r^2\cos^2\varphi + r^2\sin^2\varphi = \underline{\underline{r^2}}$$

$$\lim_{x,y\to 0,0} \frac{\overbrace{x^2 y}^{f}}{x^2+y^2} = 0$$

$$\frac{(r\cos\varphi)^2 r\sin\varphi}{r^2\cos^2\varphi + r^2\sin^2\varphi} =$$

$$= \frac{\cancel{r^2}\cdot r\cdot\cos^2\varphi\cdot\sin\varphi}{\cancel{r^2}} = r\cos^2\varphi\sin\varphi$$

$$\underline{\underline{-r}} \le r\cos^2\varphi\sin\varphi \le \underbrace{\underline{\underline{r}}}_{g(r)}$$

$$0 \le \lim_{\substack{r\to 0_+ \\ \varphi\in b}} r\cos^2\varphi\sin\varphi \le 0$$