5. cvičení

http://www.karlin.mff.cuni.cz/∼kuncova/
kytaristka@gmail.com

Hinty

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2} \qquad \lim_{x\to 0}\frac{\arctan\, x}{x}=1$$

$$\lim_{x\to \infty} x\,\mathrm{arccot}\, x=1$$

$$\lim_{x\to 0}\frac{\tan\, x}{x}=1 \qquad \lim_{x\to 0}\frac{\arcsin\, x}{x}=1 \qquad \lim_{x\to 1-}\frac{\arccos\, x}{\sqrt{1-x}}=\sqrt{2}$$

Příklady

1. Spočtěte limity zadaných funkcí

(a) $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}$

(b) $\lim\limits_{x\to 0}\dfrac{\sin 3x^2}{x^2}$

(c) $\lim\limits_{x\to 0}\dfrac{\mathrm{tg}\, x}{x}$

(d) $\lim\limits_{x\to 0}\dfrac{x^4}{1-\cos 4x^2}$

(e) $\lim\limits_{x\to 0+}\dfrac{\mathrm{tg}\,\sqrt{x}}{\sqrt{2x}}$

(f) $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\cos x^2}}{1-\cos x}$

(g) $\lim\limits_{x\to \infty}\arcsin\dfrac{1-x}{1+x}$

(h) $\lim\limits_{x\to 0}\ln\left(\dfrac{x}{\sin x}\right)$

(i) $\lim\limits_{x\to \infty}\arccos\left(\sqrt{x^2+x}-x\right)$

(j) $\lim\limits_{x\to 0} x\,\mathrm{cotg}\, 3x$

(k) $\lim\limits_{x\to \infty}\dfrac{\sin x}{x}$

(l) $\lim\limits_{x\to 0}\dfrac{\mathrm{tg}\, x-\sin x}{\sin^3 x}$

(m) $\lim\limits_{x\to 0}\dfrac{\sin 5x-\sin 3x}{\sin x}$

(n) $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 3x}{x^2}$

(o) $\lim\limits_{x\to \frac{\pi}{4}}\mathrm{tg}\, 2x\mathrm{tg}\,\left(\dfrac{\pi}{4}-x\right)$

(p) $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$

(q) $\lim\limits_{x\to \frac{\pi}{6}}\dfrac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

(r) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\mathrm{tg}\, x}-\sqrt{1+\sin x}}{x^3}$

(s) $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}$

(t) $\lim\limits_{x\to \pi}\dfrac{\sin mx}{\sin nx}$, kde $m,n\in\mathbb{N}$
(Hint: použijte "substituci" $y=x-\pi$.)

(u) $\lim\limits_{x\to \infty}\dfrac{\arctan\, x}{\mathrm{arccot}\, x}$

Zkouškové příklady

2. Spočtěte limity zadaných funkcí

   (a) $\lim\limits_{x\to 0} \dfrac{\sin(\tan\, x)}{\arctan\, (\arcsin\, x)}$

   (b) $\lim\limits_{x\to 0} \dfrac{1-\cos(\arctan\, x)}{x^2}$

   (c) $\lim\limits_{x\to \infty} x^{5/2}\arcsin\, (\sqrt{x^5+1}-\sqrt{x^5-1})$

   (d) $\lim\limits_{x\to \frac{3\pi}{2}} (4x^2-9\pi^2)\dfrac{\cos x}{1+\sin x}$

   (e) $\lim\limits_{x\to \infty} \dfrac{\arctan\, (\sqrt{x^2+\sin^2 x}-\sqrt{x^2-\cos^2 x})}{\sqrt{x^2+2}-\sqrt{x^2+1}}$

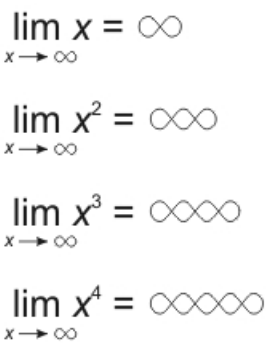

Figure 1: https://mathjokes4mathyfolks.wordpress.com/tag/limits/