

## 6. cvičení

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### Teorie

**Věta 1.** Nechť  $a_n$  je kladná posloupnost. Nechť navíc existuje

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = A.$$

Pak existuje i

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = A.$$

### Hinty

$$A^2 - B^2 = (A - B)(A + B)$$

$$\cos(n\pi) = (-1)^n$$

### Příklady

Vyšetřete konvergenci řad. (Všechna  $x, p, q, \alpha \in \mathbb{R}$ .)

1.  $\sum_{n=1}^{\infty} \frac{1}{n \operatorname{arccotg} \sqrt{2n}}$
2.  $\sum_{n=1}^{\infty} \tan \frac{n^2}{2^n}$
3.  $\sum_{n=1}^{\infty} \frac{(\sqrt{n+2} - \sqrt{n}) \ln(n^2 + n)}{n^2}$
4.  $\sum_{n=1}^{\infty} \cos(n\pi) \ln \frac{n^2 - 1}{n^2 + 1}$
5.  $\sum_{n=1}^{\infty} \left( n \arcsin \frac{1}{2n} \right)^n$
6.  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 2}$
7.  $\sum_{n=1}^{\infty} \sin \frac{x}{\sqrt{n} \ln n}$
8.  $\sum_{n=1}^{\infty} \sin^2 \frac{x}{\sqrt{n} \ln n}$
9.  $\sum_{n=1}^{\infty} \arctan \left( \frac{e^n}{e^n + 1} \right) \ln \left( \frac{e^n - 1}{e^n + 1} \right) \cos n$
10.  $\sum_{n=1}^{\infty} \frac{n+1}{n+3} \left( \ln \frac{n+3}{n+1} \right)^n$
11.  $\sum_{n=1}^{\infty} \frac{n! + 1}{(n+2)! + 2}$
12.  $\sum_{n=1}^{\infty} \frac{\binom{n}{2} + \binom{n}{3}}{\binom{n}{4} + \binom{n}{5}}$
13.  $\sum_{n=1}^{\infty} \left( 1 - \ln \left( \frac{n+1}{n} \right) \right)^{n^2}$
14.  $\sum_{n=1}^{\infty} \left( \frac{1+2^n}{3^n} + \frac{\sqrt{n+2} - \sqrt{n+1}}{\sqrt[4]{n}} \right)$
15.  $\sum_{n=1}^{\infty} \left( \cos \frac{x}{n} \right)^n$
16.  $\sum_{n=1}^{\infty} \frac{n^p + 1}{n^q + n^2 - 3}$

$$17. \sum_{k=1}^{\infty} \frac{x^k}{1+x^{2k}}$$

$$18. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$$

$$19. \sum_{k=1}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$$

$$20. \sum_{k=1}^{\infty} \cos(k^2\pi) \left( \sqrt{k+9} - \sqrt{k} \right)$$

$$21. \sum_{n=0}^{\infty} \frac{(n!)^\alpha}{(2n)!}$$

$$22. \sum_{k=1}^{\infty} (-1)^k \left( \sqrt[k]{\frac{k^2}{k^2+1}} - 1 \right).$$

**Bonus**