$y^{(n)}(x) + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = 0$

$y^{(4)} - 3y' - y = 0$ $\quad \in \mathbb{R}$

$y''' - 6y'' + 9y' = 0$ $\quad 3y \to 5$

char. pol.

$\lambda^3 - 6\lambda^2 + 9\lambda = 0$

$\lambda(\lambda - 3)^2 = 0$

$\lambda_1 = 0 \qquad \lambda_2 = 3$ ← 2-nás.

$y_1 = e^{0x} \qquad y_2 = e^{3x} \qquad y_3 = xe^{3x}$

$y = c_1 e^{0x} + c_2 e^{3x} + c_3 x e^{3x}$ $\quad c_{1,2,3} \in \mathbb{R}$, $x \in \mathbb{R}$

$y'' = y \qquad y = c_1 e^x + c_2 e^{-x}$

$y'' = -y \qquad y = c_1 \sin x + c_2 \cos x$

$y^{(4)} + 8y'' + 16y = 0$ $\quad$ ~~$16y \quad y\ \lambda = 1$~~

$\lambda^4 + 8\lambda^2 + 16 = 0$

$(\lambda^2 + 4)^2 = 0$

$\lambda_{12} = \dfrac{0 \pm \sqrt{0 - 16}}{2}$

$\lambda_1 = 2i$ ← 2-nás $\qquad \lambda_2 = -2i$ ← 2-nás

$0 + 2i \qquad 0 - 2i$

$(\lambda - 2i)(\lambda + 2i)(\lambda - 2i)(\lambda + 2i)$

$\alpha + \beta i \quad \alpha - \beta i \longrightarrow e^{\alpha x}\cos \beta x$

$e^{\alpha x} \sin \beta x$

$y_1 = e^{0x}\cos(2x) \qquad y_2 = e^{0x}\sin(2x)$

$y_3 = xe^{0x}\cos(2x) \qquad y_4 = xe^{0x}\sin(2x)$

~~$y_3 = x^2$~~ ~~x^3~~

$[e^{2ix} \qquad e^{-2ix}]$

$y = c_1 e^{0x}\cos(2x) + c_2 e^{0x}\sin(2x) + c_3 x e^{0x}\cos(2x) + c_4 x e^{0x}\sin(2x)$

$c_{1-4} \in \mathbb{R} \qquad x \in \mathbb{R}$

$y'' = -y \qquad y'' + y = 0 \qquad y = c_1 \sin x + c_2 \cos x$

$y(0) = 2 \qquad y'(0) = 3$

$y(0) = c_1 \sin 0x + c_2 \cos 0 = 2$

$y'(x) = c_1 \cos x + c_2(-\sin x)$

$c_1 \cos 0 + c_2(-\sin 0) = 3$

$\boxed{\begin{aligned} c_2 &= 2 \\ c_1 &= 3 \end{aligned}}$

$y = 3\sin x + 2\cos x$

$$\boxed{y'' + 9y} = \underline{8\cos 1x}$$

$y'' + 9y = 0$

$\lambda^2 + 9 = 0$

$\lambda_1 = -3i \quad \lambda_2 = +3i$

$y_H = C_1 \cos 3x + C_2 \sin 3x$

$$e^{\mu x}\left(P(x)\cos(\nu x) + Q(x)\sin(\nu x)\right)$$

$\mu, \nu \in \mathbb{R}$ $\quad$ P, Q polynomy

$$e^{0x}\left(8\cos(1x) + 0\sin(1x)\right)$$

$\mu = 0 \qquad \nu = 1$

$P(x) = 8$ st. $= 0 \qquad Q(x) = 0$ st. $= 0$

$$y_P = x^m e^{\mu x}\left(R(x)\cos(\nu x) + S(x)\sin(\nu x)\right)$$

$$y_P = x^0 e^{0x}\left(A\cos(1x) + B\sin(1x)\right)$$

R, S pol. 0-tého st. $\qquad$ m: $\underline{\mu + \nu i = 0 + i}$ → byl to kořen

Nebyl. → $m = 0$

ODHAD

$y_P = x^0 e^{0x}(A\cos x + B\sin x)$

$= A\cos x + B\sin x$

$y_P' = -A\sin x + B\cos x$

$y_P'' = -A\cos x - B\sin x$

$y'' + 9y = 8\cos x$

$$-A\cos x - B\sin x + 9(A\cos x + B\sin x) = 8\cos x$$

$$8A\cos x + 8B\sin x = 8\cos x + 0\sin x$$

$8A = 8 \qquad 8B = 0$

$A = 1 \qquad B = 0$

$$\boxed{y_P} = \cos x + 0\sin x = \boxed{\cos x}$$

$y = y_H + y_P$

$$y = C_1\cos 3x + C_2\sin 3x + \cos x$$

$C_1, C_2 \in \mathbb{R} \qquad x \in \mathbb{R}$

$$y'' + y = \underbrace{x}_{f} + \underbrace{\sin x}_{g} \checkmark$$

$$y'' + y = 0 \longrightarrow y_H = C_1 \_\_\_\_ + C_2 \_\_\_\_$$

$$y'' + y = x \qquad y_{p_1} = \_\_\_\_$$

$$y'' + y = \sin x \qquad y_{p_2} = \sin x \quad \cos x$$

$$y = y_H + y_{p_1} + y_{p_2}$$