$\int |x| \, dx$ $\quad |x|$ spoj. na $\mathbb{R}$

$\quad \hookrightarrow$ má PF

$$|x| = \begin{cases} x & x > 0 \\ 0 & x = 0 \\ -x & x < 0 \end{cases}$$

$$\int x = \frac{x^2}{2} + C_1 \qquad x \in (0, \infty)$$

$$\int -x = -\frac{x^2}{2} + C_2 \qquad x \in (-\infty, 0)$$

v 0: chceme 1 spoj. funkci

$$\lim_{x \to 0-} -\frac{x^2}{2} + C_2 = C_2$$

$\qquad\qquad\qquad \parallel$ (chceme)

$$\lim_{x \to 0+} \frac{x^2}{2} + C_1 = C_1$$

$-\frac{x^2}{2} + C_2 \qquad 0 \qquad \frac{x^2}{2} + C_1$

$\longrightarrow \quad C_1 = C_2$

$$F(x) = \begin{cases} -\frac{x^2}{2} + C_1 & x \in (-\infty, 0) \\ C_1 & x = 0 \\ \frac{x^2}{2} + C_1 & x \in (0, \infty) \end{cases}$$

$C_1 \in \mathbb{R}$

F spoj v 0 $\qquad$ (F spoj.)

F' existuje na $(0, \infty)$ $\qquad$ Věta

$$\lim_{x \to 0+} \left(\frac{x^2}{2} + C_1\right)' = \lim_{x \to 0+} x = 0 = F'_+(0)$$

f spoj.

zleva

$$\lim_{x \to 0-} \left(-\frac{x^2}{2} + C_2\right)' = \lim_{x \to 0-} -x = 0 = F'_-(0)$$

$F'_+(0) = 0 = F'_-(0) \longrightarrow F'(0) \ \exists$:)

potřebujeme $F'(0) = f(0) = |0|$:)

$\int |\sin x|$

$x \in \mathbb{R}$ spoj. na $\mathbb{R}$

$\longrightarrow$ PF na $\mathbb{R}$

$$|\sin x| = \begin{cases} \sin x & x \in (0 + 2k\pi, \pi + 2k\pi) \\ -\sin x & x \in (-\pi + 2k\pi, 0 + 2k\pi) \end{cases}$$

$$\int \sin x \, dx = -\cos x + c_{\underline{k}}$$

$$x \in (0 + 2\underline{k}\pi, \pi + 2\underline{k}\pi)$$

$$\int -\sin x \, dx = \cos x + d_k$$

$$x \in (-\pi + 2k\pi, 0 + 2k\pi)$$

$-\pi \qquad 0 \qquad \pi \qquad 2\pi$

$c_{-1} \qquad d_0 \qquad c_0 \qquad d_1 \qquad c_1 \quad \dots$

fix 1 bod: 0

$$\lim_{x \to 0+} -\cos x + c_0 = -1 + c_0$$

$$\lim_{x \to 0-} \cos x + d_0 = 1 + d_0$$

$$-1 + c_0 = 1 + d_0$$

fix $\underline{c_0}$ $\qquad \boxed{d_0 = c_0 - 2}$

další bod: π

$$\lim_{x \to \pi-} -\cos x + c_0 = 1 + c_0$$

$$\lim_{x \to \pi+} \cos x + d_1 = -1 + d_1$$

$$1 + \underline{c_0} = -1 + d_1$$

$$\boxed{c_0 + 2 = d_1}$$

další bod: $-\pi$

$$\lim_{x \to -\pi-} -\cos x + c_{-1} = 1 + c_{-1}$$

$$\lim_{x \to -\pi+} \cos x + d_0 = -1 + d_0 = -1 + c_0 - 2$$

$$1 + c_{-1} = -1 + c_0 - 2$$

$$c_{-1} = c_0 - 4$$

$-2\pi \qquad -\pi \qquad 0 \qquad \pi \qquad 2\pi$

$\underline{-\cos x + c_0}$ -4 $\qquad \cos x + c_0$ $\underline{-2}$ $\qquad \underline{-\cos x + c_0}$ $\qquad \cos x + c_0 +$ $\underline{2}$ $\qquad \underline{-\cos x + c_0}$ $+4$

$$F(x) = \begin{cases} \underline{-\cos x + c_0} + \cancel{2k} \;\; \underline{4k} & x \in (0 + 2k\pi, \pi + 2k\pi) \\ -1 + c_0 + 4k & \longrightarrow x = 0 + 2k\pi \\ 1 + c_0 + 4k & \longrightarrow x = \pi + 2k\pi \\ \underline{\cos x + c_0 + 4k - 2} & x \in (-\pi + 2k\pi, 0 + 2k\pi) \end{cases}$$

$$1 + c_0 + 4k - 2$$

$$-1 + c_0 + \cancel{4k - 2} \quad 4(k+1) - 2$$

(2b) $\displaystyle\int \frac{1}{\underbrace{2\sin x - \cos x + 5}_{f \text{ spojitá na } \mathbb{R}}}\,dx$

$x \in \mathbb{R}$

test $\to$ $t = \tan\frac{x}{2}$ $\qquad dx = \frac{2}{1+t^2}dt$

$x \in (-\pi + 2k\pi, \pi + 2k\pi)$

$-3\pi \quad -\pi \quad 0 \quad \pi \quad 3\pi$

$$\int \frac{1}{2\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} + 5}\,\frac{2}{1+t^2}dt =$$

$$\int \frac{2}{\frac{4t - 1 + t^2 + 5 + 5t^2}{\cancel{1+t^2}}}\cdot\frac{1}{\cancel{1+t^2}}dt =$$

$$= \int \frac{2}{6t^2+4t+4}dt = \int \frac{1}{3t^2+2t+2}dt$$

$$= \frac{1}{3}\int \frac{1}{t^2+\frac{2}{3}t+\frac{2}{3}}dt = \frac{1}{3}\int \frac{1}{(t+\frac{1}{3})^2+\frac{5}{9}}dt =$$

$$= \frac{1}{3}\frac{1}{\frac{5}{9}}\int \frac{1}{\left(\frac{t+\frac{1}{3}}{\frac{\sqrt{5}}{3}}\right)^2+1}dt = \frac{3}{5}\frac{\sqrt{5}}{3}\arctan\frac{3t+1}{\sqrt{5}} + C_k$$

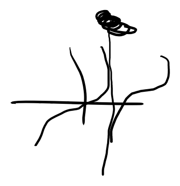
$$= \frac{1}{\sqrt{5}}\arctan\frac{3t+1}{\sqrt{5}} + C_k$$

$$= \frac{1}{\sqrt{5}}\arctan\frac{3\tan(\frac{x}{2})+1}{\sqrt{5}} + C_k$$

$x \in (-\pi + 2k\pi, \pi + 2k\pi)$

$\pi + 2k\pi$

$\frac{1}{\sqrt{5}} \ldots + C_k \qquad \ldots + C_{k+1}$



bod $\pi + 2k\pi$

$$\lim_{x\to(\pi+2k\pi)^-} \frac{1}{\sqrt{5}}\arctan\frac{3\tan(\frac{x}{2})+1}{\sqrt{5}} + C_k = \text{"}\frac{1}{\sqrt{5}}\arctan\frac{\infty+1}{\sqrt{5}} + C_k\text{"}$$

$$= \frac{1}{\sqrt{5}}\frac{\pi}{2} + C_k$$

$$\lim_{x\to(\pi+2k\pi)^+} \frac{1}{\sqrt{5}}\arctan\Big(\underbrace{\frac{3\tan\frac{x}{2}+1}{\sqrt{5}}}_{-\infty}\Big) + C_{k+1} = \frac{-1}{\sqrt{5}}\frac{\pi}{2} + C_{k+1}$$

$$\frac{\pi}{2\sqrt{5}} + C_k = \frac{-\pi}{2\sqrt{5}} + C_{k+1} \qquad \boxed{C_{k+1} = C_k + \frac{\pi}{\sqrt{5}}}$$

$-\pi + 2\cdot 0\pi \quad \pi + 2\cdot 0\pi \quad \pi + 2\cdot 1\pi \quad \pi + 2\cdot 2\pi$

$C_{-2} \quad C_{-1} \quad +C_0 \quad +C_1 \quad +C_2$

fix C_0 $\qquad C_{-1} = C_0 - \frac{\pi}{\sqrt{5}} \qquad C_1 = C_0 + \frac{\pi}{\sqrt{5}} \qquad C_2 = C_1 + \frac{\pi}{\sqrt{5}} = C_0 + 2\cdot\frac{\pi}{\sqrt{5}}$

$$F(x) = \begin{cases} \frac{1}{\sqrt{5}}\arctan\frac{3\tan\frac{x}{2}+1}{\sqrt{5}} + \overbrace{C_0 + \frac{k\pi}{\sqrt{5}}}^{C_k} & x \in (-\pi + 2k\pi, \pi + 2k\pi) \\ \frac{\pi}{2\sqrt{5}} + \underbrace{C_0 + \frac{k\pi}{\sqrt{5}}}_{+C_k} & x = \pi + 2k\pi \end{cases}$$

1. Věta o subst.