$$\frac{xy + y^2 - 3x^3}{x - yx^7}$$

$$\int \frac{\sin x \cos x + \cos^2 x - 3\sin^3 x}{\sin x - \cos x \sin^7 x}\, dx$$

$$\int \frac{\sin x}{1 - (\cos x)^2}\, dx \qquad x \neq k\pi \quad k \in \mathbb{Z}$$

$-2\pi \quad -\pi \quad 0 \quad \pi \quad 2\pi$

(1) $\dfrac{\sin x}{1 - (-\cos x)^2} = \dfrac{\sin x}{1 - \cos^2 x}$ $\qquad (0, \pi)$ $\quad \alpha, \beta$ $\quad (k\pi, (k+1)\pi)$

(2) $\dfrac{-\sin x}{1 - (\cos x)^2} = - \dfrac{\sin x}{1 - (\cos x)^2}$

$t = \cos x$

(3) $\dfrac{-\sin x}{1 - (-\cos x)^2} = - \dfrac{\sin x}{1 - \cos^2 x}$

$[t = \cos x \qquad dt = -\sin x\, dx$ $\qquad \varphi(x) = \cos x$

$\varphi'(x) = -\sin x$

$$\int \underbrace{\frac{-1}{1 - t^2}}_{f} dt = -\frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| + c$$

(α, β)

$(0, \pi)$

dicono $k\pi \quad (k+1)\pi$

$$= -\frac{1}{2} \ln \left| \frac{1 + \cos x}{1 - \cos x} \right| + c$$

$\varphi((0, \pi))$

$k\pi \quad (k+1)\pi$

$= (-1, 1)$

$f(t) = \dfrac{-1}{1 - t^2}$ $\qquad t \neq \pm 1$

$~~(-\infty, -1)~~ \qquad \boxed{(-1, 1) \quad (a, b)} \qquad ~~(1, \infty)~~$

$(k\pi, (k+1)\pi)$

$$\int \frac{1}{\sin^2 x + 2\cos^2 x}\,dx \qquad x \in \mathbb{R}$$

$$\frac{1}{(-\sin x)^2 + 2(-\cos x)^2} = \frac{1}{\sin^2 x + 2\cos^2 x} \checkmark$$

$\varphi = \tan x$

$\varphi' = \frac{1}{\cos^2 x}$

$\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$

(3) $\boxed{t = \tan x}$ $\quad dt = \frac{1}{\cos^2 x}\,dx$

$$\sin^2 x = \frac{\tan^2 x}{1+\tan^2 x} \qquad \frac{\frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$$

$$\cos^2 x = \frac{1}{1+\tan^2 x}$$

$$\int \frac{1}{\frac{\tan^2 x}{1+\tan^2 x} + 2\cdot\frac{1}{1+\tan^2 x}} \cdot \frac{\frac{1}{1+\tan^2 x}}{\cos^2 x}\,dx$$

(α, β)

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$+ k\pi$

$$= \int \frac{1}{\frac{t^2}{1+t^2} + \frac{2}{1+t^2}} \cdot \left(\frac{1}{1+t^2}\right) dt$$

$\varphi \left(\subseteq \mathbb{R}\right)$

$$= \int \frac{1}{\frac{t^2+2}{\cancel{1+t^2}}} \cdot \frac{1}{\cancel{1+t^2}}\,dt =$$

$$= \boxed{\int \frac{1}{2+t^2}} dt = \int \frac{1}{2\left(1+\left(\frac{t}{\sqrt{2}}\right)^2\right)} dt$$

F

$t \in \mathbb{R} = (a, b)$

$$= \frac{1}{2}\sqrt{2} \arctan \frac{t}{\sqrt{2}} + c$$

$$= \frac{\sqrt{2}}{2} \arctan\left(\frac{\tan x}{\sqrt{2}}\right) + c$$

$\arctan(\tan x) \neq x$

$x \neq \frac{\pi}{2} + k\pi$

$-\pi/2 \qquad \pi/2 \qquad \frac{3\pi}{2}$

(α, β)

$\sin^2 x$

$1 - \cos^2 x$

$$\frac{1 - \cos 2x}{2}$$

$\sin^2 x$

$- \cos^2 x$

$$\frac{- \cos 2x}{2}$$

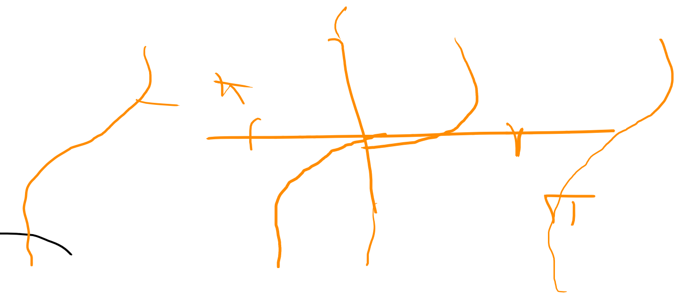
$$\int \frac{1}{2+\sin x}\,dx \qquad f(\varphi)\cdot\varphi' \qquad (2)\ \frac{1}{2-\sin x}$$

$\rightarrow x \in \mathbb{R}$

$\varphi(x)$

$\operatorname{tg}\frac{x}{2} = t \qquad (-\pi, \pi) + 2k\pi \qquad (\alpha, \beta)$

dx



$$\int \frac{1}{2+\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt =$$

$\varphi' = \frac{1}{\cos^2\frac{x}{2}} \cdot \frac{1}{2}$

$\varphi(\alpha,\beta) = \mathbb{R}$ ✓

$$= \int \frac{1}{2t^2+2t+2} \cdot 2\,dt =$$

$$= \int \frac{1}{t^2+t+1}\,dt = \int \frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}\,dt$$

$f(t) \rightarrow$

$t \in \mathbb{R}$

(a, b)

$$= \frac{2\sqrt{3}}{2} \arctan \frac{2t+1}{\sqrt{3}} + C$$

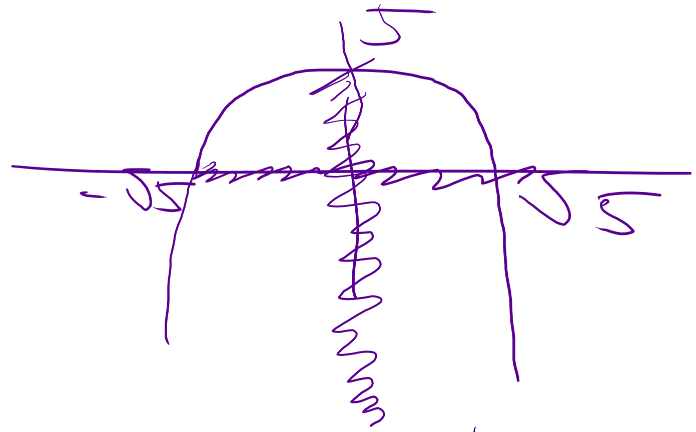
$$= \sqrt{3} \arctan \frac{2\operatorname{tg}\frac{x}{2}+1}{\sqrt{3}} + C$$

$x \in (-\pi, \pi) + 2k\pi$

$\sqrt{y}$

$\sqrt{5-x^2}$

$-x^2+5$



vnitřní

D_f zmenšit

$\rightarrow$ H_f zmenšit

nejde Dreižu

$$\underbrace{\frac{1}{2+\frac{2t}{1+t^2}}\cdot\frac{2}{1+t^2}}_{f}$$

$f(\varphi)$

$$\frac{1}{2+\frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}}\cdot\frac{2}{1+\operatorname{tg}^2\frac{x}{2}}$$

$$=\frac{1}{2+\sin x}\cdot\frac{1}{2\cos^2\frac{x}{2}}$$