$$\int \frac{2x^4+2x^2-5x+1}{x(x^2-x+1)^2}\,dx =$$

$D = 1-4\cdot1\cdot1 = -3$ nemá kořeny

stupně: $4<5$ ✓

$$= \int \frac{A}{x} + \frac{Bx+C}{x^2-x+1} + \frac{Dx+E}{(x^2-x+1)^2}\,dx$$

$$2x^4+2x^2-5x+1 = A(x^2-x+1)^2 + (Bx+C)x(x^2-x+1) + x(Dx+E)$$

$$= A(x^4-x^3+x^2-x^3+x^2-x+x^2-x+1)$$

$$+ B(x^4-x^3+x^2) + C(x^3-x^2+x)$$

$$+ Dx^2 + Ex$$

x^4: $2 = A+B$

x^3: $0 = -2A - B + C$

x^2: $2 = 3A + B - C + D$

x: $-5 = -2A + C + E$

1: $1 = A$

$A = 1$ $\quad$ $2 = 1 + B$

$B = 1$ $\quad$ $0 = -2 - 1 + C$

$C = 3$

$E = -6$

$D = 1$

$-5 = -2 + 3 + E$

$-6 = E$

$2 = 3 + 1 - 3 + D$

$$= \int \frac{1}{x} + \frac{1x+3}{x^2-x+1} + \frac{x-6}{(x^2-x+1)^2}\,dx$$

$\ln|x|$

$$\frac{1}{2}\int \frac{2x-1+1+6}{x^2-x+1}\,dx = \frac{1}{2}\ln|x^2-x+1|$$

chceme $2x-1$

$$\frac{1}{2}\int \frac{2x-1}{x^2-x+1}\,dx$$

$$+\frac{1}{2}\int \frac{7}{x^2-x+1}\,dx$$

$$\frac{7}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx$$

$$= \frac{7}{2}\int \frac{1}{\frac{3}{4}} \cdot \frac{1}{\left(\frac{x-\frac{1}{2}}{\sqrt{\frac{3}{4}}}\right)^2+1}$$

$$= \frac{14}{3}\int \frac{1}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}\,dx$$

$$= \frac{14}{3}\cdot\frac{\sqrt{3}}{2}\arctan\frac{2x-1}{\sqrt{3}}$$

$$\int \frac{x-6}{(x^2-x+1)^2}\,dx = \frac{1}{2}\int \frac{2x-1+1-12}{(x^2-x+1)^2}\,dx$$

chcemo $2x-1$

$$= \frac{1}{2}\int \frac{2x-1}{(x^2-x+1)^2}\,dx + \boxed{-\frac{11}{2}\int \frac{1}{(x^2-x+1)^2}\,d}$$

$$u = x^2 - x + 1$$

$$du = 2x-1\,dx$$

$$\frac{1}{2}\int \frac{1}{u^2}\,du = \frac{1}{2}\cdot -\frac{1}{u} = -\frac{1}{2}\,\frac{1}{x^2-x+1}$$

$$-\frac{11}{2}\int\frac{1}{(x^2-x+1)^2}dx = -\frac{11}{2}\int\frac{1}{\left(\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right)^2}dx$$

$$= -\frac{11}{2}\int\frac{1}{\left(\frac{3}{4}\right)^2}\cdot\frac{1}{\left(\left(\frac{x-1/2}{\sqrt{\frac{3}{4}}}\right)^2+1\right)^2}dx$$

$$= \frac{-8\cdot 11}{9}\int\frac{1}{\left(\left(\frac{2x-1}{\sqrt{3}}\right)^2+1\right)^2}dx =$$

$$y=\frac{2x-1}{\sqrt{3}} \quad dy=\frac{2}{\sqrt{3}}dx$$

$$= \frac{-88}{9}\int\frac{\frac{\sqrt{3}}{2}}{(y^2+1)^2}dy$$

$$\int\frac{1}{(y^2+1)^n}dy = \frac{1}{2n-2}\frac{y}{(y^2+1)^{n-1}}$$

$$n=2 \qquad + \frac{2n-3}{2n-2}\int\frac{1}{(y^2+1)^{n-1}}dy$$

$$\underbrace{-\frac{44\sqrt{3}}{9}}_{z}\int\frac{1}{(y^2+1)^2}dy =$$

$$z\left(\frac{1}{4-2}\frac{y}{(y^2+1)^{2-1}}+\frac{4-3}{4-2}\int\frac{1}{(y^2+1)^{2-1}}dy\right.$$

$$\overset{C}{=} z\cdot\left(\frac{1}{2}\frac{y}{(1+y^2)^1}+\frac{1}{2}\arctan y\right)$$

$$= \frac{-44\sqrt{3}}{9}\left(\frac{1}{2}\frac{\frac{2x-1}{\sqrt{3}}}{\left(1+\left(\frac{2x-1}{\sqrt{3}}\right)^2\right)}+\frac{1}{2}\arctan\frac{2x-1}{\sqrt{3}}\right)$$

celkem pro $\int$

$$\overset{C}{=} \ln|x| + \frac{1}{2}\ln|x^2-x+1| + \frac{7}{3}\sqrt{3}\arctan\frac{2x-1}{\sqrt{3}}$$

$$+ -\frac{1}{2}\frac{1}{x^2-x+1}$$

$$+ \frac{-44\sqrt{3}}{9}\left(\frac{1}{2}\frac{\frac{2x-1}{\sqrt{3}}}{1+\left(\frac{2x-1}{\sqrt{3}}\right)^2}+\frac{1}{2}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

$x \neq 0$

(1)

$$\int \frac{1}{(1+y^2)^n}\,dy = \int \frac{1+y^2}{(1+y^2)^n} + \frac{-y^2}{(1+y^2)^n}\,dy$$

$$= \int \frac{1}{(1+y^2)^{n-1}}\,dy + \int -\frac{1}{2}\, y \,\frac{2y}{(1+y^2)^n}\,dy$$

$$u = y \qquad v' = \frac{2y}{(1+y^2)^n}$$

$$u' = 1$$

$$v = \frac{1}{1-n}\,\frac{1}{(1+y^2)^{n-1}}$$

$$-\frac{1}{2}\,\frac{y}{(1-n)(1+y^2)^{n-1}} - \left(-\frac{1}{2}\right)\int \frac{1}{1-n}\cdot\frac{1}{(1+y^2)^{n-1}}\,dy$$

dohromady

$$= +\frac{y}{(2n-2)(1+y^2)^{n-1}} + \left(1 + \frac{1}{2-2n}\right)\int \frac{1}{(1+y^2)^{n-1}}\,dy$$

2)

$$\int \frac{1}{(1+y^2)^n}\,dy \qquad y=\tan t$$

$$dy=\frac{1}{\cos^2 t}\,dt$$

$$\int \frac{1}{(1+\tan^2 t)^n}\cdot\frac{1}{\cos^2 t}\,dt =$$

$$=\int \frac{1}{\left(1+\frac{\sin^2 t}{\cos^2 t}\right)^n}\cdot\frac{1}{\cos^2 t}\,dt =$$

$$=\int \frac{(\cos^2 t)^n}{1}\cdot\frac{1}{\cos^2 t}\,dt =$$

$$=\int (\cos t)^{2n-2}\,dt$$

Möli pine $n=2$

$$\int (\cos t)^{4-2}\,dt=\int \cos^2 t\,dt$$

$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$\cos^2 t+\sin^2 t=1$$

$$\cos^2 t-\sin^2 t=\cos 2t$$

$$\int \frac{1}{2}+\frac{1}{2}\cos 2t\,dt =$$

$$= \frac{1}{2}t+\frac{1}{4}\underbrace{\sin 2t}_{\frac{1}{2}\sin t\cos t} \qquad \frac{\tan t}{1+\tan^2 t}$$

$$t=\arctan y$$

$$=\frac{1}{2}\arctan y+\frac{1}{4}\sin 2\arctan y$$

$$=\frac{1}{2}\arctan y+\frac{1}{2}\,\frac{y}{1+y^2}$$