$$\int \frac{2x+3}{(x-2)(x+5)}\,dx$$

$$\int x = \frac{x^2}{2} + c \qquad \int \frac{1}{2x+3} = \frac{1}{2}\ln|2x+3| + C$$

$$\int \frac{1}{(-3x+7)^2} = \int (-3x+7)^{-2} = -\frac{1}{3}\,\frac{(-3x+7)^{-1}}{-1} + c$$

$$\int \frac{1}{1+(4x)^2} = \frac{\arctan 4x}{4} + c$$

$$\int \frac{1}{1+(x-3)^2}\,dx = \arctan(x-3) + c$$

$$\int \frac{2x+3}{x^2+3x+6}\,dx = \int \frac{1}{y}\,dy = \ln|y| + c$$

$$y = x^2+3x+6 \qquad = \ln|x^2+3x+6| + C$$

$$dy = 2x+3\,dx$$

$$\int \frac{2x+3}{(x-2)(x+5)}\,dx = \int \frac{1}{x-2} + \frac{1}{x+5}\,dx = \ln|x-2| + \ln|x+5| + c$$

$x \neq 2 \quad x \neq -5$

$$\frac{x+5+x-2}{(x-2)(x+5)}$$

$x \in (-\infty,-5)$

$x \in (-5,2)$

$x \in (2,\infty)$

$$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} \rightarrow \frac{1}{x-2} + \frac{1}{x+5}$$

Kolik je A a B

$$\frac{2x+3}{(x-2)(x+5)} = \frac{A(x+5)+B(x-2)}{(x-2)(x+5)}$$

$$2x+3 = A(x+5) + B(x-2)$$

$x = 2$: $\quad 4+3 = A(2+5) + B(2-2)$

$7 = 7A \qquad \boxed{A = 1}$

$x = -5$: $\quad -7 = A(-5+5) + B\cdot(-7)$

$\boxed{1 = B}$

$$\int \frac{x}{x^3-3x+2}\,dx = \int \frac{x}{(x-1)(x^2+x-2)} = \int \frac{x}{(x-1)(x-1)\cdot(x+2)}\,dx$$

Problém: jmenovatel: x, x^2 Problém 2 jsou kořeny

když $x=1$ $\quad 1^3-3\cdot 1+2=0$

$$(x^3-3x+2):(x-1)=x^2+x-2$$
$$-(x^3-x^2)$$
$$x^2-3x+2$$
$$-(x^2-x)$$
$$-2x+2$$
$$-(-2x+2)$$
$$0$$:)

$$\int \frac{x}{(x+2)(x-1)^2}\,dx = \int \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

(A: $-2/9$; B: $2/9$; C: $1/3$)

$$\frac{x}{(x+2)(x-1)^2} = \frac{A(x-1)^2+B(x+2)(x-1)+C(x+2)}{(x+2)(x-1)^2}$$

$$x = A(x-1)^2+B(x+2)(x-1)+C(x+2)$$

$x=1$: $\quad 1 = A\cdot 0 + B\cdot 0 + C(1+2)$

$$1/3 = C$$

$x=-2$ $\quad -2 = A(-3)^2 + B\cdot 0 + C\cdot 0$

$$-2/9 = A$$

$x=0$ $\quad 0 = A\cdot 1 + B(-2) + 2C$

$$0 = -\frac{2}{9} - 2B + \frac{2}{3}$$

$$0 = -2B + \frac{4}{9}$$

$$B = \frac{2}{9}$$

$$0x^2+1x+0 = A(x^2-2x+1) + B(x^2+x-2) + C(x+2)$$

$$0x^2+1x+0 = x^2(A+B) + x(-2A+B+C) + A-2B+2C$$

$$\int \frac{1/3}{(x-1)^2} + \frac{2/9}{x-1} + \frac{-2/9}{x+2}\,dx = \frac{1}{3}\frac{(x-1)^{-1}}{-1} + \frac{2}{9}\ln|x-1| - \frac{2}{9}\ln|x+2| + c$$

$x \neq 1 \quad x \neq -2$

$$0 = A+B$$
$$1 = -2A+B+C$$
$$0 = A-2B+2C$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -2 & 1 & 1 & 1 \\ 1 & -2 & 2 & 0 \end{array}\right)$$

:)

$$\int \frac{1}{(x^2-4x+4)(x^2-4x+5)}\,dx$$

$$= \int \frac{1}{(x-2)^2(x^2-4x+5)}\,dx$$

$$= \int \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2-4x+5}$$

$$1 = A(x-2)(x^2-4x+5) + B(x^2-4x+5) + Cx(x-2)^2 + D(x-2)^2$$

$x=2$ $\quad 1 = B(4-8+5)$ $\quad \boxed{B=1}$

$x=0$

$x=1$

$$1 = -10A + 5\cdot 1 + 4D$$

$$1 = A(-1)(-2) + 1\cdot 2 + C\cdot 1\cdot 1 + D\cdot 1$$

$x=-1$

$$1 = A(-3)10 + \cancel{D}\cdot 10 + C(-1)\cdot 9 + D\cdot 9$$

$D=-1$ $\quad A=0$ $\quad C=0$

$$\int \frac{1}{(x-2)^2} + \frac{-1}{x^2-4x+5}\,dx$$

$$\frac{(x-2)^{-1}}{-1}$$

$$\int \frac{-1}{(x-2)^2+1}$$

$$-1\arctan(x-2)$$

$$= \frac{-1}{x-2} - \arctan(x-2) + C$$

$x \neq 2$

$$\int \frac{x^6+2x-1}{x^5-x^2}\,dx = \int x + \frac{x^3+2x-1}{x^5-x^2}$$

$$\frac{P}{Q} \qquad st\,P < st\,Q$$

$$(x^6+2x-1) : (x^5-x^2) = x$$

$$-(x^6-x^3)$$

$$x^3+2x-1$$

$$x^6-x^3+x^3+2x-1$$

$$x^5-x^2$$

$$\int x + \frac{x^3+2x-1}{x^2(x^3-1)}\,dx = \int x + \frac{x^3+2x-1}{x^2(x-1)\cdot(x^2+x+1)}$$

$$\frac{x^3+2x-1}{x^2(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{B}{x} + \frac{C}{x^2} + \frac{Dx+E}{x^2+x+1}$$

$$\int x + \frac{2/3}{x-1} + \frac{-2}{x} + \frac{1}{x^2} + \frac{\frac{4}{3}x + \frac{2}{3}}{x^2+x+1}\,dx$$

$$\frac{2x+1}{x^2+x+1} \to \ln|x^2+x+1|$$

$$\frac{2}{3}\cdot\frac{2x+1}{x^2+x+1}$$

$$= \frac{x^2}{2} + \frac{2}{3}\ln|x-1| - 2\ln|x| - \frac{1}{x} + \frac{2}{3}\ln|x^2+x+1|$$

$$x \neq 0 \qquad x \neq 1$$

$$\int \frac{x^2+2x-7}{(x-1)(x^2-4x+5)}\,dx$$

(1) st P < st Q ? $2 < 3$ ✓

(2) nelze rozložit ✓

(3) $= \int \frac{A}{x-1} + \frac{Bx+C}{x^2-4x+5}\,dx$

$x^2+2x-7 = A(x^2-4x+5) + Bx\cdot(x-1) + C(x-1)$

$= \int \frac{-2}{x-1} + \frac{3x-3}{x^2-4x+5}$

$-2\ln|x-1|$

chceme $\frac{2x-4}{x^2-4x+5}$

$\frac{3}{2}\int \frac{2x-2}{x^2-4x+5}$

$= \frac{3}{2}\int \frac{2x-4+4-2}{x^2-4x+5}\,dx$

$= \frac{3}{2}\int \frac{2x-4}{x^2-4x+5}\,dx + \frac{3}{2}\int \frac{2}{x^2-4x+5}\,dx$

$\frac{3}{2}\ln|x^2-4x+5|$

$y = x^2-4x+5$

$dy = 2x-4\,dx$

$\frac{3}{2}\int \frac{1}{y}\,dy$

$= \ln|\dots|$

$3\int \frac{1}{(x-2)^2+1}$

$3\arctan(x-2)$

$= -2\ln|x-1| + \frac{3}{2}\ln|x^2-4x+5| + 3\arctan(x-2) + C$

$x + + 1$

$$\int \frac{1}{x^2-4x+7}\,dx = \int \frac{1}{(x-2)^2+3} =$$

$$= \int \frac{1}{3\left(\frac{(x-2)^2}{3}+1\right)}\,dx =$$

$$= \frac{1}{3}\int \frac{1}{\left(\frac{x-2}{\sqrt{3}}\right)^2+1}\,dx =$$

$$= \frac{1}{3}\sqrt{3}\arctan\frac{x-2}{\sqrt{3}} + C$$

☺