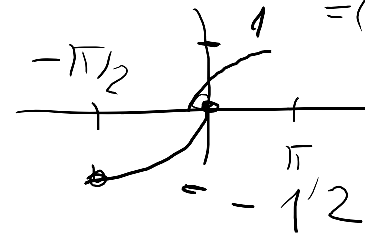
$$\int f(\varphi)\cdot\varphi' = G$$

$$\to \quad \int f(x) = G(\varphi^{-1}(x))$$

$\varphi(\alpha,\beta) = (-1, 1)$ $= (a,b)$



$f(x)$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx \overset{c}{=} \arcsin x$$

$t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

α, β

$x \in (-1,1)$

(a,b)

$\varphi(t)$

$x = \sin t$ $\quad \varphi'(t)$ $\quad \underbrace{\arcsin x}_{\varphi^{-1}(x)} = t$

$dx = \boxed{\cos t \;\; dt}$ $\quad \cos t \neq 0$ na (α,β)

$$\int \frac{1}{\sqrt{1-(\sin t)^2}} \cos t\, dt \overset{!}{=} \int \frac{\cos t}{\cancel{\cos t}}\, dt$$

$f(\varphi(t))$ $\quad \varphi'(t)$

$G(t)$

$$= \int 1\, dt \overset{c}{=} \boxed{t}$$

$$= \arcsin x \; \} G(\varphi^{-1})$$

$$\sinh x = \frac{e^{x}-e^{-x}}{2} \qquad \cosh x = \frac{e^{x}+e^{-x}}{2} \qquad x\in\mathbb{R}$$

$$(\sinh x)' = \cosh x \qquad (\cosh x)' = \sinh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$\cosh^2 x = 1 + \sinh^2 x$

$x\in\mathbb{R}$:-)

$$\int \sqrt{4+x^2}\,dx = \int 2\sqrt{1+\frac{x^2}{4}}\,dx$$

$x = 2\sinh \cancel{x}\, t \qquad \frac{x}{2} = \sinh t$

$dx = 2\cosh t\,dt \qquad \operatorname{argsinh}\left(\frac{x}{2}\right) = t$

$$= \int \sqrt{4+4\sinh^2 t}\cdot 2\cosh t\,dt =$$

$$= 4\int \sqrt{1+\sinh^2 t}\cdot \cosh t\,dt = 4\int \sqrt{\cosh^2 t}\cdot \cosh t\,dt$$

$$= 4\int \cosh^2 t\,dt \stackrel{\subseteq}{=} 2\Big(t + \underbrace{(\cosh t)\cdot(\sinh t)}_{\frac{1}{2}\sinh(2t)}\Big)$$

$$= 2\left(\operatorname{argsinh}\frac{x}{2} + \frac{1}{2}\sinh\left(2\operatorname{argsinh}\frac{x}{2}\right)\right)$$:-)

$$= 2\left(\ln \frac{x}{2} + \sqrt{\left(\frac{x}{2}\right)^2+1} + \frac{1}{2}\,\frac{e^{+(2\operatorname{argsinh}\frac{x}{2})} - e^{-(2\operatorname{argsinh}\frac{x}{2})}}{2}\right.$$

Vsuvka

$$\int \cos^2 t \, dt \longrightarrow \int \cos t \cdot \cos t \, dt$$

2 Per Partes

$$\cos^2 t \overset{?}{=} \frac{1+\cos 2t}{2}$$

$$\cos^2 t + \sin^2 t = 1$$

$$\cos^2 t - \sin^2 t = \cos 2t$$

$$2\cos^2 t = 1 + \cos 2t$$

$\cosh^2 t :$ $\rightarrow$ 2 per partes

$\rightarrow \frac{1}{2}(e^{x} + e^{-x})$

$\rightarrow$ vzorec

$$\cosh^2 t - \sinh^2 t = 1$$

$$\cosh^2 t + \sinh^2 t = \cosh(2t)$$

$$\int \frac{1}{1+\sqrt{x}} \, dx$$

$$\int \underbrace{\frac{1}{1+\sqrt{x}}}_{f(\varphi)} \cdot \underbrace{\frac{2\sqrt{x}}{2\sqrt{x}}}_{\varphi'} \, dx$$

1. Věta: $f(\varphi)$ $\quad f = \frac{2y}{1+y}$ $\quad \varphi = \sqrt{x}$

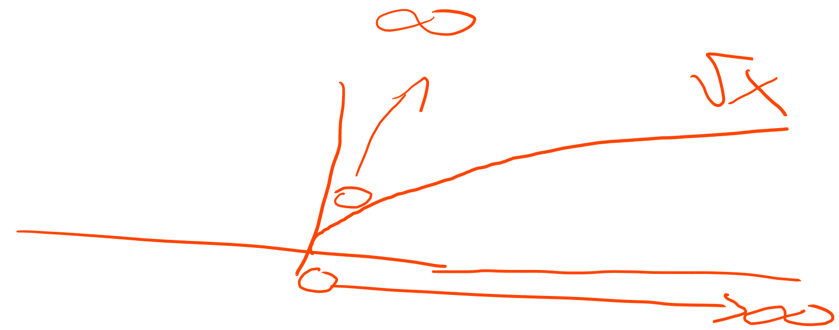
$\varphi' = \frac{1}{2\sqrt{x}}$

$$y = \sqrt{x}$$

$$dy = \frac{1}{2\sqrt{x}} \, dx$$

$x > 0$

$f: \; y \in \mathbb{R} \setminus \{-1\}$



$$= \int \frac{1}{1+y} \cdot 2y \, dy \qquad y \in (-1, \infty)$$

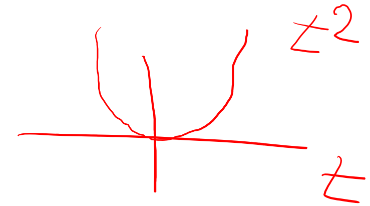
$\varphi: (0, \infty) \xrightarrow{do} (-1, \infty)$

$$= 2 \int \frac{\overbrace{y+1}^{1}}{1+y} + \frac{-1}{1+y} \, dy$$

$$\overset{c}{=} 2(y - \ln|1+y|)$$

$$= 2(\sqrt{x} - \ln(1+\sqrt{x}))$$

$$\int \underbrace{\frac{1}{1+\sqrt{x}}}_{f\ x\in(0,\infty)}\,dx$$



2. Věta: a, b

$$\boxed{\cancel{\sqrt{\ }}x = \overset{\varphi(t)}{t^2}} \qquad \left(\sqrt{x} = t \longrightarrow x = t^2\right)$$

$$dx = \underset{\varphi'(t)}{2t}\,dt \qquad t = \sqrt{x} \leftarrow \varphi^{-1}(x)$$

$$= \int \frac{1}{1+\sqrt{t^2}} \cdot 2t\,dt = \qquad \neq 0 \qquad \cancel{t\in\mathbb{R}} \quad t\in(0,\infty)$$

ma :) $\qquad (\alpha, \beta)$

$$\varphi((0,\infty)) = (0,\infty)$$
$$(a, b)$$

$$= \int \frac{1}{1+t}\,2t\,dt$$

$$\overset{c}{=} 2\left(t - \ln|1+t|\right)$$

$$= 2\left(\sqrt{x} - \ln(1+\sqrt{x})\right)$$