Substituce

Věta 1 (o substituci)**.** *Nechť $G \subset \mathbb{R}^n$ je otevřená množina a $\varphi : G \to \mathbb{R}^n$ je prosté regulární zobrazení. Nechť u je funkce na $M \subset \varphi(G)$. Potom*

$$\int_M u(x)\,dx = \int_{\varphi^{-1}(M)} u(\varphi(t))|J\varphi(t)|\,dt,$$

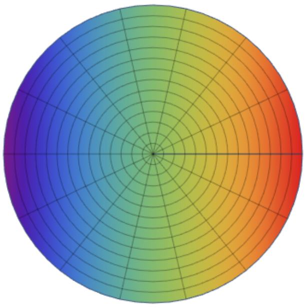
pokud alespoň jedna strana má smysl.

Polární souřadnice

$$x(r,\alpha) := r\,\cos\alpha,$$
$$y(r,\alpha) := r\,\sin\alpha$$

$$r > 0,$$
$$-\pi < \alpha < \pi,$$

$$J\varphi(r,\alpha) = r.$$

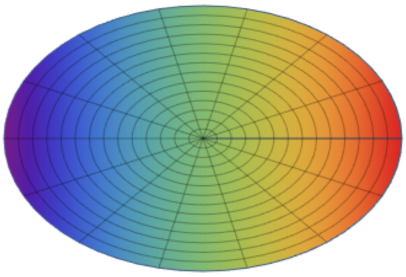


Zobecněné polární souřadnice

$$x(r,\alpha) := x_0 + ar\,\cos\alpha,$$
$$y(r,\alpha) := y_0 + br\,\sin\alpha$$

$$r > 0,$$
$$-\pi < \alpha < \pi,$$

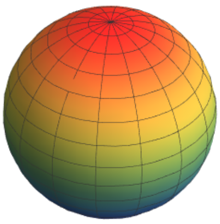
$$J\varphi(r,\alpha) = abr.$$



Sférické souřadnice

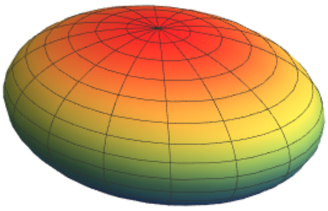
$$x(r,\beta,\gamma) := r\,\cos\gamma\,\cos\beta,$$
$$y(r,\beta,\gamma) := r\,\cos\gamma\,\sin\beta,$$
$$z(r,\beta,\gamma) := r\,\sin\gamma$$
$$r > 0,$$
$$-\pi < \beta < \pi,$$
$$-\frac{\pi}{2} < \gamma < \frac{\pi}{2},$$

$$J_\varphi = r^2\cos\gamma$$



Zobecněné sférické souřadnice

$$x(r,\beta,\gamma) := x_0 + ar\,\cos\gamma\,\cos\beta,$$
$$y(r,\beta,\gamma) := y_0 + br\,\cos\gamma\,\sin\beta,$$
$$z(r,\beta,\gamma) := z_0 + cr\,\sin\gamma$$

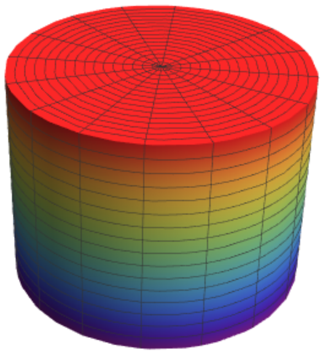


$$r > 0,$$
$$-\pi < \beta < \pi,$$
$$-\frac{\pi}{2} < \gamma < \frac{\pi}{2},$$

$J_\varphi = abcr^2 \cos\gamma$

Cylindrické souřadnice

$$x(r,\alpha,z) := r\,\cos\alpha$$
$$y(r,\alpha,z) := r\,\sin\alpha,$$
$$z(r,\alpha,z) := z$$

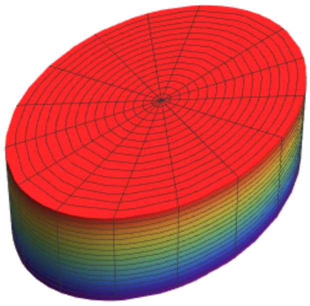


$$r > 0,$$
$$-\pi < \alpha < \pi,$$
$$z \in \mathbb{R}$$

$J_\varphi = r$

Zobecněné cylindrické souřadnice

$$x(r,\alpha,z) := x_0 + ar\,\cos\alpha$$
$$y(r,\alpha,z) := y_0 + br\,\sin\alpha,$$
$$z(r,\alpha,z) := z_0 + z$$



$$r > 0,$$
$$-\pi < \alpha < \pi,$$
$$z \in \mathbb{R}$$

$J_\varphi = abr$