(2a)

$$y' = \begin{pmatrix} 4 & -2 & -4 \\ 4 & -2 & -3 \\ 2 & -2 & -2 \end{pmatrix} y$$

$$\begin{pmatrix} \lambda-4 & 2 & 4 \\ -4 & \lambda+2 & 3 \\ -2 & 2 & \lambda+2 \end{pmatrix} \begin{matrix} \cdot \lambda+2 \\ + \\ \cdot -2 \end{matrix} \sim \begin{pmatrix} \lambda-4 & 2 & 4 \\ (\lambda-4)(\lambda+2)+8 & 0 & 4\lambda+2 \\ 2-\lambda & 0 & \lambda-2 \end{pmatrix} \begin{matrix} 1\cdot(\lambda-2) \\ + \\ 1\cdot(4\lambda+2) \end{matrix}$$

SPATNE

$* \quad \cdot 1(-4)$

$$\sim \begin{pmatrix} \lambda-4 & 2 & 4 \\ \lambda^2-2\lambda & 0 & 4\lambda+2 \\ (\lambda^2-2\lambda)(2-\lambda)+(2-\lambda)(4\lambda+2) & 0 & 0 \end{pmatrix}$$

$$y_1 = c_1 e^{2x} + c_2 e^{-x}\cos x + c_3 e^{-x}\sin x$$

$$(2-\lambda)\left[\lambda^2-2\lambda+4\lambda+2\right] = 0$$

$$(2-\lambda)(\lambda^2+2\lambda+2)$$

$$\lambda_1 = 2 \qquad \lambda_2 = -1+i \qquad \lambda_3 = -1-i$$

$$* \begin{pmatrix} \lambda-4 & 2 & 4 \\ -8+4\lambda+\lambda^2-2\lambda & 0 & 10 \\ 2-\lambda & 0 & \lambda-2 \end{pmatrix}$$

$$10y_3 = -(\lambda^2+2\lambda-8)y_1$$

$$10y^3 = -y_1'' - 2y' + 8y_1$$

$\vdots$

$$y^3 = c_2 e^{-x}\cos x + c_3 e^{-x}\sin x$$

$$2y_2 = -4y_3 - (y_1' - 4y_1)$$

$$2y_2 = -4(c_2 e^{-x}\cos x + c_3 e^{-x}\sin x) - (c_1 e^{2x} + c_2 e^{-x}\cos x + c_3 e^{-x}\sin x)' + 4(c_1 e^{2x} + c_2 e^{-x}\cos x + c_3 e^{-x}\sin x)$$

pak $y_2 = \frac{1}{2} e^{-x}\left((c_2+c_3)\sin x + (c_2-c_3)\cos x\right) + c_1 e^{2x}$

(2a) $y' = \begin{pmatrix} 4 & -2 & -4 \\ 4 & -2 & -3 \\ 2 & -2 & -2 \end{pmatrix} y$

$$-\left(\begin{pmatrix} \lambda-4 & 2 & 4 \\ -4 & \lambda+2 & 3 \\ -2 & 2 & \lambda+2 \end{pmatrix}\right)^{\cdot(\lambda+2)}_{\cdot(-2)} \sim \begin{pmatrix} \lambda-4 & 2 & 4 \\ (\lambda+2)(\lambda-4)+8 & 0 & 4\lambda+2 \\ \lambda-2 & 0 & -\lambda+2 \end{pmatrix} \sim$$

(jen rozepsání)

$$\sim \begin{pmatrix} \lambda-4 & 2 & 4 \\ \lambda(\lambda-2) & 0 & 2(2\lambda+1) \\ \lambda-2 & 0 & 2-\lambda \end{pmatrix}_{\cdot(-\lambda)} \sim \begin{pmatrix} \lambda-4 & 2 & 4 \\ \lambda(\lambda-2) & 0 & 2(2\lambda+1) \\ 0 & 0 & -\lambda(2-\lambda)+2(2\lambda+1) \end{pmatrix}$$

y_3: $-2\lambda+\lambda^2+4\lambda+2$

$\lambda^2+2\lambda+2=0$

$\lambda=-1\pm i$

$y_3 = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

y_1: $y_1' - 2y_1 = -2y_3 + y_3'$

$y_1' - 2y_1 = e^{-x}\left((-3c_1+c_2)\cos x + (-c_1-3c_2)\sin x\right)$

spec PS

$-1+1i$ není kořen

$\lambda-2=0$ $\quad y_{1h} = c_3 e^{2x}$

$\lambda=2$

$y_{1p} = e^{-x}(A\cos x + B\sin x)$

pak $e^{-x}\left((-3A+B)\cos x + (-A-3B)\sin x\right) = e^{-x}\left((-3c_1+c_2)\cos x + (-c_1-3c_2)\sin x\right)$

zřejmě $A=c_1$, $B=c_2$

pak $y_1 = c_3 e^{2x} + c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

nakonec $y_2 = \frac{1}{2}\left(2y_1 - y_3' - 2y_3\right)$

$y_2 = \frac{1}{2}e^{-x}\left((c_1+c_2)\sin x + (c_1-c_2)\cos x + 2c_3 e^{3x}\right)$

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(2b)

$$y' = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & 2 \\ 3 & 1 & 4 \end{pmatrix} \cdot y$$

$$\begin{array}{ccc} y_1 & y_2 & y_3 \end{array}$$

$$\begin{pmatrix} \lambda-1 & -1 & 2 \\ -1 & \lambda-1 & -2 \\ -3 & -1 & \lambda-4 \end{pmatrix} \begin{matrix} \cdot \lambda-4 \\ + \\ \cdot 2 \end{matrix} \sim \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ 4-\lambda-6 & (\lambda-1)(\lambda-4)-2 & 0 \end{pmatrix} \sim$$

$$\sim * \left(+ \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ -2-\lambda & \lambda^2-5\lambda+2 & 0 \end{pmatrix} \right. \begin{matrix} \cdot(2+\lambda) \\ + \\ \cdot(\lambda-2) \end{matrix} \sim \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ 0 & (\lambda^2-4)+(\lambda^2-5\lambda+2)\cdot(\lambda-2) & 0 \end{pmatrix}$$

$= -(2+\lambda)$

ŠPATNĚ

$$\lambda^2-4+\lambda^3-5\lambda^2+2\lambda-2\lambda^2+10\lambda-4 = \lambda^3-6\lambda^2+12\lambda-8 = (\lambda-2)^3$$

3-nás. kořen $\lambda = 2$

$$\underline{y_2 = c_1 e^{2x} + c_2 x e^{2x} + c_3 x^2 e^{2x}}$$

$$* \sim \begin{pmatrix} \lambda-1 & -1 & 2 \\ -4 & \lambda^2-4\lambda & 0 \\ -2-\lambda & \lambda^2-5\lambda+2 & 0 \end{pmatrix}$$

pak $+4y_1 = y_2'' - 4y_2'$

$$+4y_1 = -2e^{2x}\left(2c_1 + 2c_2 x + c_3(2x^2-1)\right)$$

$$y_1 = -e^{2x}\left(\underline{c_1 + c_2 + c_3(x^2 - \tfrac{1}{2})}\right)$$

$$-2y_3 = y_1' - 1y_1 - y_2$$

$$-2y_3 = -\tfrac{1}{2}e^{2x}\left(4c_1 + 2c_2 x + 2c_2 + 4c_3 x^2 + 4c_3 x - c_3\right)$$

$$y_3 = e^{2x}\left(c_1 + \tfrac{c_2}{2}x + \tfrac{c_2}{2} + c_3 x^2 + c_3 x - \tfrac{c_3}{4}\right)$$

(2b) $y' = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & 2 \\ 3 & 1 & 4 \end{pmatrix} y$

$$\begin{array}{l} (\lambda-4)\cdot \\ + \\ (2) \end{array} \left(\begin{pmatrix} \lambda-1 & -1 & 2 \\ -1 & \lambda-1 & -2 \\ -3 & -1 & \lambda-4 \end{pmatrix} \right)+ \sim \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ -\lambda-2 & (\lambda-1)(\lambda-4)-2 & 0 \end{pmatrix} \sim$$

$$\sim \begin{array}{l} \\ \\ +(\end{array} \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ -\lambda-2 & \lambda^2-5\lambda+2 & 0 \end{pmatrix} \sim \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ -4 & \lambda^2-4\lambda & 0 \end{pmatrix} \begin{array}{l} \\ )\cdot 4 \\ + \\ \cdot(\lambda-2) \end{array}$$

$$\sim \begin{pmatrix} \lambda-1 & -1 & 2 \\ \lambda-2 & \lambda-2 & 0 \\ 0 & \begin{array}{c}(\lambda^2-4\lambda)(\lambda-2) \\ +4(\lambda-2)\end{array} & 0 \end{pmatrix}$$

y_2:

$$\lambda^3 - 2\lambda^2 - 4\lambda^2 + 8\lambda + 4\lambda - 8 = 0$$
$$\lambda^3 - 6\lambda^2 + 12\lambda - 8 = 0$$
$$(\lambda-2)^3 = 0$$

3-nás koreň $\lambda = 2$

$$y_2 = c_1 e^{2x} \underline{+ c_2 x e^{2x}} + c_3 x^2 e^{2x}$$

y_1:

$$4y_1 = y_2'' - 4y_2'$$
$$4y_1 = -2e^{2x}(2c_1 + 2c_2 x + c_3(2x^2-1))$$
$$y_1 = -e^{2x}(c_1 + c_2 + c_3(x^2 - \tfrac{1}{2}))$$

y_3:

$$-2y_3 = y_1' - y_1 - y_2$$
$$y_3 = e^{2x}(c_1 + \tfrac{c_2}{x}x + \tfrac{c_2}{2} + c_3 x^2 + c_3 x - \tfrac{c_3}{4})$$