(1) $\displaystyle\int_1^{\infty} \frac{x^5+32}{(x^6-1)\sqrt[3]{x}}\,dx$

u 1: $g(x) = 1/x-1$ $\qquad \displaystyle\int_1^{42} \frac{1}{x-1}\,dx = \infty$

$$\lim_{x\to 1+} \frac{(x-1)\overbrace{(x^5+32)}^{\to 33}}{\underbrace{(x^6-1)}_{\text{zL'H}\ \lim_{x\to 1+}\frac{1}{6x^5}=\frac{1}{6}}\ \underbrace{\sqrt[3]{x}}_{\to 1}} = \frac{33}{6} \qquad \int_1^{42} f \text{ div zLS2}$$

u ∞: $g(x) = \dfrac{x^5}{x^6\sqrt[3]{x}} = \dfrac{1}{x\sqrt[3]{x}}$ $\qquad \displaystyle\int_2^{\infty} x^{-4/3} < \infty$

$$\lim_{x\to\infty} \frac{x^5+32}{x^5}\cdot\frac{x^6}{x^6-1}\cdot\frac{\sqrt[3]{x}}{\sqrt[3]{x}} = 1 \qquad \text{z LS2 } \int_2^{\infty} f \ \underline{\underline{k}}$$

(2) $\displaystyle\int_0^{\pi} \frac{x+4}{\left(x-\frac{5\pi}{2}\right)\sqrt{x\sin x}}\,dx$

u 0: $\quad g(x) = \dfrac{1}{\sqrt{x^2}} \overset{\text{na } (0,\pi)}{=} \dfrac{1}{x} \qquad \displaystyle\int_0^{\pi/2} \frac{1}{x} = \infty$

$$\lim_{x\to 0+} \frac{(x+4)\sqrt{x^2}}{\left(x-\frac{5\pi}{2}\right)\sqrt{x\sin x}} = -\frac{8}{5\pi} \qquad \int_0^{\pi/2} f \ \text{div} \qquad \text{z LSK}$$

u π: $\quad g(x) = \dfrac{1}{\sqrt{-x+\pi}} \qquad \displaystyle\int_{\pi/2}^{\pi} \frac{1}{\sqrt{\pi-x}} < \infty$

$$\lim_{x\to\pi-} \frac{\sqrt{-x+\pi}}{\sqrt{\sin x}} = \sqrt{1}$$

$$\lim_{x\to\pi-} \frac{\pi-x}{\sin x} \overset{\text{L'H}}{\underset{0/0}{=}} \lim_{x\to\pi-} \frac{-1}{\cos x} = 1$$

$$\lim_{x\to\pi-} \frac{f}{g} = \frac{\pi+4}{\pi-\frac{5\pi}{2}} \quad \text{vlastní limita}$$

$$\int_{\pi/2}^{\pi} f < \infty \quad \text{z LSK}$$

(3)

(a) $$\int_{-\infty}^{-1} \frac{x+2}{(x+1)\ln(1-\sqrt[3]{x})}\,dx$$

u $-\infty$: $\quad g(x) = 1/\ln -x$

$$\lim_{x\to-\infty} \frac{(x+2)\ln -x}{(x+1)\ln(1-\sqrt[3]{x})} = \lim \underbrace{\frac{x+2}{x+1}}_{\downarrow\, 1} \cdot \underbrace{\lim_{x\to\infty} \frac{\ln -x}{\ln(1-\sqrt[3]{x})}}_{\downarrow\, \text{L'H}} = 3$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{1-\sqrt[3]{x}}\cdot\left(-\frac{1}{3}\right)\frac{1}{x^{2/3}}} = \lim_{x\to\infty} \frac{-3\,x^{2/3}(1-\sqrt[3]{x})}{x} = 3$$

$$\int_{-\infty}^{-2} \frac{1}{\ln(-x)}\,dx = \int_2^{\infty} \frac{1}{\ln y}\,dy = \infty \text{ (z tabulky)}$$

$y=-x \quad dy = -dx$

tedy z LSK $\int_{-\infty}^{-2} f = \infty$

u -1: $\quad g(x) = \frac{1}{x+1} \qquad \int_{-2}^{-1} \frac{1}{x+1} = \infty$

$$\lim_{x\to-1-} \frac{(x+2)(x+1)}{(x+1)\ln(1-\sqrt[3]{x})} = \frac{1}{\ln 2}$$

z LSK $\int_{-2}^{-1} f = \infty$

(b) $\int_{-1}^{0} f$

u -1 div z (a)

u 0: $\quad g(x) = \frac{1}{-\sqrt[3]{x}} \qquad \int_{-1/2}^{0} -\frac{1}{\sqrt[3]{x}}\,dx < \infty$

$$\lim_{x\to 0-} \frac{(x+2)(-\sqrt[3]{x})}{(x+1)\ln(1-\sqrt[3]{x})} = 2$$

z LSK $\int_{-1/2}^{0} f$ <u>konv</u>

(4) $\displaystyle\int_{-\infty}^{0} \frac{e^x \ln|x|}{x(x-1)}\,dx$

u $-\infty$: $\displaystyle\int_{-\infty}^{-2} \frac{e^x \ln|x|}{x(x-1)}\,dx = \int_{2}^{\infty} \frac{e^{-y}\ln y}{-y(-y-1)}\,dy$

$y=-x \quad dy=-dx$

$$\frac{e^{-y}|\ln y|}{y(y+1)} \underset{\substack{\uparrow\\ \text{SK}}}{\leq} \frac{e^{-y}}{y} \qquad \int_2^{\infty} \frac{e^{-y}}{y}\,dy \quad \underline{\text{konv.}}$$

ze SK $\displaystyle\int_2^{\infty} \frac{e^{-y}|\ln y|}{y(y+1)} < \infty$

u 0: $\displaystyle\int_{-2}^{0} \frac{e^x \ln|x|}{x(x-1)}\,dx = \int_0^2 \frac{e^{-y}\ln y}{+y(y+1)}\,dy$

$y=-x \quad dy=-dx$

$g(y) = \dfrac{\ln y}{y} = \ln y \cdot y^{-1} \qquad \displaystyle\int_0^2 |\ln y|\,\frac{1}{y}\,dy = \infty$

tabulka

$\displaystyle\lim_{y\to 0+} \frac{e^{-y}\cancel{\ln y}\cdot \cancel{y}}{\cancel{y}(y+1)\cdot\cancel{\ln y}} = 1 \qquad$ z LSK $\displaystyle\int_{-2}^{0} f\,dx = \underline{\infty}$

$\displaystyle\int_0^1 \frac{e^x \ln|x|}{x(x-1)}\,dx$

u 0 jako v (a)

u 1 lze spoj. dodef $\rightarrow$ $\int_{1/2}^{1} f$ $\underline{\text{konv.}}$

(5) $$\int_0^1 \frac{1}{\arcsin(\sin \pi x)\cdot\sqrt{|\ln(\frac{1}{2}+|x|)|}}\,dx$$

v 0: $g(x) = \frac{1}{x}$ $\qquad \int_0^{1/2} \frac{1}{x} = \infty$

$$\lim_{x\to 0+} \frac{x}{\arcsin(\sin \pi x)\sqrt{|\ln(\frac{1}{2}+|x|)|}} = \frac{1}{\pi\sqrt{|\ln 2|}}$$

$$\lim_{x\to 0+} \frac{\sin \pi x}{\arcsin \pi x}\cdot\frac{\pi x}{\sin \pi x}\cdot\frac{1}{\pi} = \frac{1}{\pi}$$

z LSK $\int_0^{1/2} f = \infty$

v 1: $\sin \pi x \approx \pi - \pi x = \pi(1-x)$ $\qquad \int_{1/2}^1 \frac{1}{\pi(1-x)}\,dx = \infty$

$g(x) = 1/\pi(1-x)$

$$\lim_{x\to 1-} \frac{\pi(1-x)}{\arcsin(\sin \pi x)} \overset{l'H}{=} \lim_{x\to 1-} \frac{-\pi}{\sqrt{1-(\sin \pi x)^2}}\cdot\frac{1}{\cos \pi x \cdot \pi}$$

$$= \frac{-\pi}{-\pi} = 1$$

$$\lim_{x\to 1-} \frac{f}{g} = \frac{1}{\sqrt{\ln 3/2}}$$

z LSK $\int_{1/2}^1 f$ $\underline{\underline{\text{div}}}$

(6)

(a) $\displaystyle\int_{-\infty}^{-1} \frac{\arctan(x+1)}{\underbrace{\sqrt[3]{\arctan x}\,(1-\arctan x)}_{f(x)}}\,dx$

$\underline{\underline{-\infty}}$: $\arctan x,\ \arctan(x+1) \approx -\frac{\pi}{2}$

tedy fce se chová jako konstanta $\quad g(x) = \dfrac{-\pi/2}{-\frac{\pi}{2}\left(1+\frac{\pi}{2}\right)} = \dfrac{1}{1+\frac{\pi}{2}}$

$$\lim_{x\to-\infty} \frac{\dfrac{\arctan(x+1)}{\sqrt[3]{\arctan x}\,(1-\arctan x)}}{1+\pi/2} = 1 \qquad \int_{-\infty}^{-2} \frac{1}{1+\frac{\pi}{2}} = \infty$$

LSZ

$\to$ $f(x)$ <u>div.</u>

$\underline{\underline{-1}}$ fci lze spoj. dodefinovat $\to$ $\int_{-2}^{-1} f(x)$ konv.
(a zpět.)

(b) $\displaystyle\int_{-1}^{0} f(x)$

u 0 $\quad \arctan x \approx x \quad \to \quad$ srovnáme s $g(x) = \dfrac{\pi/4}{\sqrt[3]{x}}$

$$\lim_{x\to 0-} \frac{\dfrac{\arctan(x+1)}{\sqrt[3]{\arctan x}\,(1-\arctan x)}}{\dfrac{\pi/4}{\sqrt[3]{x}}} = 1 \qquad \int_{-1/2}^{0} \frac{\frac{\pi}{4}}{\sqrt[3]{x}} < \infty$$

z LSZ $\int_{-1/2}^{0} f(x)$ <u>konv.</u> $\qquad$ (u -1 lze spoj dodef) jako minule

(6)

(c) $\int_0^{\tan 1} f(x)$

u 0 jako výše

u tg1: $1 - \arctan x \approx$?

zkusíme $(x - \text{tg}1)^\alpha$ $\lim_{x \to \tan 1-} \frac{1-\arctan}{(x-\tan 1)^\alpha} \overset{l'H}{\underset{\frac{0}{0}}{=}} \lim_{x \to \frac{\pi}{4}-} \frac{-\frac{1}{1+x^2}}{\alpha(x-\text{tg}1)^{\alpha-1}}$

pro $\alpha = 1$ dostaneme $\lim_{x \to \text{tg}1} \frac{-1}{1+x^2} = \frac{-1}{1+\text{tg}^2 1}$

$g(x) = \frac{\arctan(\text{tg}1+1)}{(x-\text{tg}1)}$ (musí být > 0)

! $\lim_{x \to \text{tg}1-} \frac{f(x)}{g(x)} = -1-\text{tg}^2 1$ $\int_{0{,}01}^{\tan 1} \frac{\arctan(\text{tg}1+1)}{(x-\text{tg}1)} dx = \infty$

(subst. a škála)

z LSK $\to$ $\int_0^{\text{tg}1} f$ $\underline{\text{div}}$

(d) $\int_{\text{tg}1}^{\infty} \frac{\arctan(x+1)}{\sqrt[3]{\arctan x}\,(1-\arctan x)} dx$

u tg1: jako výše $\to$ div

u ∞ : jako u $-\infty$ $\to$ div

(7)

(a) $$\int_{-\infty}^{-1} \frac{1}{\underbrace{(1+x)\sqrt[3]{x(x-1)^2}}_{f}}\,dx$$

u $-\infty$: $\quad f \sim x^2$

$$\lim_{x\to-\infty} \frac{\dfrac{1}{(1+x)\sqrt[3]{x(x-1)^2}}}{\dfrac{1}{x^2}} = \lim \frac{x}{1+x} \cdot \frac{x}{\sqrt[3]{x^3-2x^2+x}} = 1$$

$$\int_{-\infty}^{-2} \frac{1}{x^2}\,dx < \infty \qquad \text{z LSK} \quad \int_{-\infty}^{-2} f(x) < \infty$$

u -1 : $\quad g(x) = \dfrac{-1}{1+x}$ $\qquad \displaystyle\int_{-2}^{-1} \frac{1}{1+x}\,dx = \infty$

(třeba z výpočtu)

$$\lim_{x\to-1-} \frac{x+1}{(1+x)\sqrt[3]{x(x-1)^2}} = \frac{-1}{\sqrt[3]{-1(-2)^2}} = \frac{1}{\sqrt[3]{4}}$$

z LSK $\displaystyle\int_{-2}^{-1} f(x)$ div.

(b) $$\int_{-1}^{0} \frac{1}{(1+x)\sqrt[3]{x(x-1)^2}}\,dx$$

u -1: div jako výše

u 0: $\quad g(x) = \dfrac{1}{\sqrt[3]{x}}$ $\qquad \displaystyle\int_{-1/2}^{0} \frac{1}{\sqrt[3]{x}}\,dx < \infty$

$$\lim_{x\to0-} \frac{f}{g} = \frac{1}{(1+0)\cdot\sqrt[3]{1}} = 1 \qquad \text{z LSK} \quad \int_{-1/2}^{0} f < \infty$$

(7)

(c) $\int_0^1 f$

u 0 jako výše

u 1: $g(x) = \frac{1}{\sqrt[3]{(x-1)^2}}$ $\int_{1/2}^{1} \frac{1}{\sqrt[3]{(x-1)^2}}\, dx < \infty$

$\lim_{x \to 1-} \frac{f}{g} = \frac{1}{2}$ z LSK $\int_{1/2}^{1} f$ konv.

(d) $\int_1^\infty f$

u 1: stejně

u ∞: $g(x) = \frac{1}{x^2}$ $\int_2^\infty \frac{1}{x^2}$ $\underline{\underline{k}}$

$\lim_{x \to \infty} \frac{f}{g} = 1$ LSK $\rightarrow$ $\int_2^\infty f$ $\underline{\underline{k}}$

(8)

(a) $$\int_{-\infty}^{0} \underbrace{\frac{1}{\sqrt{e^{|x|}-1}\; e^{x}(x-1)}}_{f} \, dx$$

u $-\infty$: $f \approx \dfrac{1}{x\, e^{x} e^{\frac{|x|}{2}}} = e^{-x+\frac{|x|}{2}} \dfrac{1}{x}$

$$\int_{-\infty}^{-1} e^{-x+\frac{|x|}{2}} \cdot \frac{1}{x} \, dx = \int_{1}^{\infty} -e^{y+\frac{|y|}{2}} \frac{1}{y} \, dx = -\int_{1}^{\infty} e^{\frac{3y}{2}} \frac{1}{y} = -\infty$$

$y = -x$

$dy = -1 dx$

$$\lim_{x \to -\infty} \frac{\cancel{e^{x}}\, x \; e^{|x|/2}}{\cancel{e^{x}}(x-1)\sqrt{e^{|x|}-1}} = 1$$ $(x-1) \to 1$

$\int_{-\infty}^{-1} f(x)$ div z LSK

u 0: $g = 1/\sqrt{|x|}$ $\int_{-1}^{0} 1/\sqrt{|x|} < \infty$

$$\lim_{x \to 0-} \frac{\sqrt{|x|}}{\sqrt{e^{|x|}-1}\; e^{x}(x-1)} = -1$$ LSK $\int_{-1}^{0} f < \infty$

(b) $\int_{0}^{1} f(x)$

u 0 jako v (a)

u 1: $g(x) = \dfrac{1}{x-1}$ $\int_{1/2}^{1} \dfrac{1}{x-1} = \infty$

$$\lim_{x \to 1-} \frac{x-1}{\sqrt{e^{|x|}-1}\; e^{x}(x-1)} = \frac{1}{e(\sqrt{e-1})}$$ z LSK $\int_{1/2}^{1} f = \infty$

(8)

(c) $\int_1^\infty \frac{1}{\sqrt{e^{|x|}-1} \quad e^x (x-1)} \, dx$

u 1: div $g(b)$

u ∞: $g(x) = \frac{1}{e^{\frac{3x}{2}} \cdot x} = e^{-\frac{3x}{2}} \cdot \frac{1}{x}$ $\qquad \int_2^\infty e^{-\frac{3x}{2}} x^{-1} < \infty$

$\lim_{x \to \infty} \frac{e^{\frac{3x}{2}} \cancel{x}}{\underbrace{\cancel{(x-1)}}_{1} e^x \sqrt{e^{|x|}-1}} = 1$ $\qquad \int_2^\infty f < \infty$