6. cvičení

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Hinty

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} \qquad \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1 \qquad \int_0^{\infty} e^{-x} x^n = n!$$

Příklady

1. Spočtěte

   (a) $\displaystyle\int_0^1 \ln x \ln(1-x)\,\mathrm{d}x$

   (b) $\displaystyle\int_0^1 \ln x \ln(1+x)\,\mathrm{d}x$

   (c) $\displaystyle\int_0^1 \ln \frac{1}{1-x}$

   (d) $\displaystyle\lim_{n\to\infty} \int_0^{\infty} \frac{\arctan nx}{1+x^3}\,\mathrm{d}x$

   (e) $\displaystyle\int_0^1 \ln \frac{1+x}{1-x}\,\mathrm{d}x = 2\ln 2$

   (f) $\displaystyle\int_0^1 \frac{1}{x} \ln \frac{1+x}{1-x}\,\mathrm{d}x = \frac{\pi^2}{4}$

   (g) $\displaystyle\lim_{n\to\infty} \int_1^{\infty} \frac{\mathrm{d}x}{\ln x + \ln n}$

   (h) $\displaystyle\int_0^{\infty} \frac{\sin x}{1+e^x}\,\mathrm{d}x$

   (i) $\displaystyle\int_0^{\infty} e^{-ax} \sin bx\,\mathrm{d}x = \frac{b}{a^2+b^2}$, $|b| < a$

   (j) $\displaystyle\lim_{n\to\infty} \int_0^{\infty} \frac{\mathrm{d}x}{(1+\frac{x}{n})^n \cdot \sqrt[n]{x}}$

   (k) $\displaystyle\lim_{n\to\infty} \int_0^{\infty} \frac{\sin \frac{x}{n}}{\left(1+\frac{x}{n}\right)^n}$

   (l) $\displaystyle\int_0^{\infty} e^{-x} \cos\sqrt{x}\,\mathrm{d}x = \sum_{n=0}^{\infty} (-1)^n \frac{n!}{(2n)!}$