2. VZOR

(1) $a \in \mathbb{R}, x \in \mathbb{R}$

$$|x^2 - a| > 1$$

- $x^2 > a$

$x^2 - a > 1$

$x^2 > 1 + a$

$|x| > \sqrt{1+a}$

$a > -1$

- $x^2 = a$

$|0| > 1$

$\emptyset$

- $x^2 < a \Rightarrow a > 0$

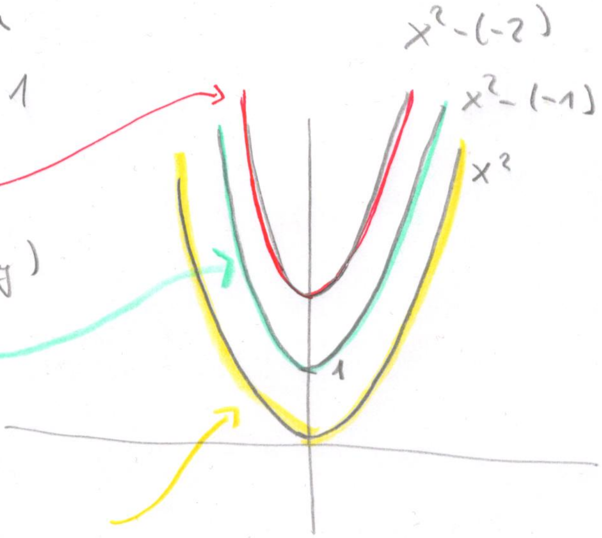
$-x^2 + a > 1$

$a - 1 > x^2$

$\sqrt{a-1} > |x|$

$\Downarrow$

$a > 1$



pro $\boxed{a < -1}$

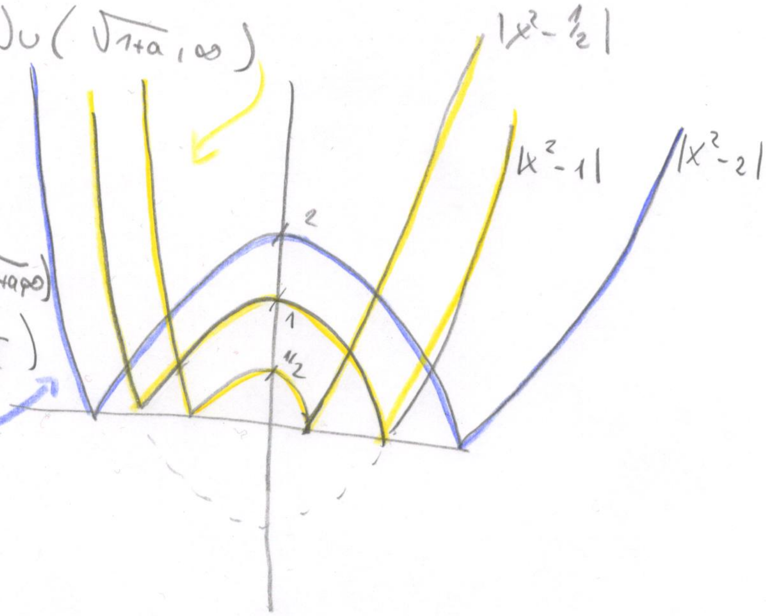
$x \in \mathbb{R}$ (automaticky)

$\boxed{a = -1}$

$x \in \mathbb{R} \setminus \{0\}$

$\boxed{-1 < a \leq 1}$

$x \in (-\infty, -\sqrt{1+a}) \cup (\sqrt{1+a}, \infty)$



$\boxed{1 < a}$

$x \in (-\infty, -\sqrt{1+a}) \cup (\sqrt{1+a}, \infty) \cup (-\sqrt{a-1}, \sqrt{a-1})$

(2) $f(x,y,z) = x^2 + 3y^2 - 2y + 3z^2$

podezřelé body : $\nabla f = 0$

tedy $\frac{\partial f}{\partial x} = 2x$ $\qquad x = 0$

$\frac{\partial f}{\partial y} = 6y - 2$ $\qquad y = 1/3$

$\frac{\partial f}{\partial z} = 6z$ $\qquad z = 0$

Podezřelý bod je $[0, 1/3, 0]$

(3) $[0,0] = a$ $\qquad e^{\sin x^2} + e^{\sin xy} = 2y + 2$

$F(x,y) = e^{\sin x^2} + e^{\sin xy} - 2y - 2 = 0$

$F: \mathbb{R}^2 \to \mathbb{R}$

Podm. : $F \in C^1(\mathbb{R}^2)$ $\quad$ (složení, součet, součin spoj. C^1 fcí)

$F(0,0) = e^{\sin 0} + e^{\sin 0} - 2\cdot 0 - 2 = 0$

$\frac{\partial F}{\partial y}(0,0) = e^{\sin xy} \cdot \cos(xy) \cdot x - 2 \Big|_{[0,0]} = -2 \neq 0$

(1) Řešení vzorcem

Pak rovnice určuje na okolí $(0,0)$ impl. zadanou fci a platí

$y'(0)$: $\quad \frac{\partial F}{\partial x} = e^{\sin x^2} \cdot \cos(x^2) \cdot 2x + e^{\sin xy} \cdot \cos(xy) \cdot y \Big|$

v $(0,0)$ $= 0$

$y'(0) = -\frac{0}{-2}$

NEBO (2) Derivací

$e^{\sin x^2} \cdot \cos x^2 \cdot 2x + e^{\sin xy(x)} \cdot \cos(xy(x)) \cdot [y(x) + xy'(x)] - 2y'(x) = 0$

$$y'(x)\left[-2+x\,e^{\sin xy(x)}\cos(xy(x))\right]=-\left(e^{\sin x^2}\cdot\cos x^2\cdot 2x+y(x)\,e^{\sin(xy(x))}\cdot\cos(xy(x))\right)$$

$$y'(x)=-\frac{e^{\sin x^2}\cdot\cos x^2\cdot 2x+y(x)\,e^{\sin(x\,y(x))}\cos(xy(x))}{-2+x\,e^{\sin xy(x)}\cos(xy(x))}$$

v 0 : $\quad y(0)=0$ (zadano)

$$y'(0)=-\frac{0+0}{-2+0}=\underline{0}$$