VZOR

$$\lim_{x\to\infty} \sqrt{\frac{x^2-2x+\frac{1}{x}}{3x^2+3}} = \underline{\underline{\frac{1}{\sqrt{3}}}}$$

(1) (a) $g(x) = \frac{x^2-2x+\frac{1}{x}}{3x^2+3}$

$f(y) = \sqrt{y}$

(b) $\lim_{x\to\infty} \frac{x^2-2x+\frac{1}{x}}{3x^2+3} = \lim \frac{x^2}{x^2} \cdot \frac{1-\frac{2}{x}+\frac{1}{x^3}}{3+\frac{3}{x^2}}$

$$\overset{\text{VOAL}}{=} \frac{\lim 1 - \lim \frac{2}{x} + \lim \frac{1}{x^3}}{\lim 3 + \lim \frac{3}{x^2}} = \frac{1-0+0}{3+0} = \frac{1}{3}$$

$\forall$ limity existují

dobře definováno

(c) $\lim_{y\to 1/3} \sqrt{y} = \sqrt{1/3}$

(d) P1 $\sqrt{y}$ spojitá v $\sqrt{1/3}$

$\lim_{y\to 1/3} f(y)$ i $\lim_{x\to\infty} g(x)$ existují

(1) (1) $\lim\limits_{x \to \infty} e^{\frac{1}{3-x}} = 1$

$g(x) = \frac{1}{3-x}$ $\lim\limits_{x \to \infty} \frac{1}{3-x} = 0$

$\lim\limits_{y \to 0} e^y = 1$

$f(y) = e^y$ P1 e^y spoj v 0

(2) $\lim\limits_{x \to -\infty} e^{-x^2} = 0$

$g(x) = -x^2$ $\lim\limits_{x \to -\infty} -x^2 = -\infty$

$f(y) = e^y$ $\lim\limits_{y \to -\infty} e^y = 0$

(P2) $g(x) \neq -\infty$ $\forall x \in \mathbb{R}$

(tedy se vyhýbá své limitě)

(3) $\lim\limits_{x \to \infty} \ln \frac{x-1}{x+1} = 0$

$g(x) = \frac{x-1}{x+1}$ $\lim\limits_{x \to \infty} \frac{x-1}{x+1} = 1$

$f(y) = \ln y$ $\lim\limits_{y \to 1} \ln y = 0$

P1, $\ln y$ spoj v 1

(4) $\lim\limits_{x \to \infty} e^{1/x} = 1$

$g(x) = 1/x$ $\lim\limits_{x \to \infty} 1/x = 0$

$f(y) = e^y$ $\lim\limits_{y \to 0} e^y = 1$

e^y spoj v 0

(1) (5) $\lim_{x\to\infty} \sin\frac{1}{x} = 0$

$g(x) = 1/x$ $\qquad \lim_{x\to\infty} 1/x = 0$

$f(y) = \sin y$ $\qquad \lim_{y\to 0} \sin y = 0$

(P1) sin spoj. v 0

(6) $\lim_{x\to 2} \sqrt{x+3} = \sqrt{5}$

$g(x) = x+3$ $\qquad \lim_{x\to 2} x+3 = 5$

$f(y) = \sqrt{y}$ $\qquad \lim_{y\to 5} \sqrt{y} = \sqrt{5}$

(P1) $\sqrt{y}$ spoj. v 5

(7) $\lim_{x\to\infty} \sqrt{\frac{x-2}{2x+1/x}} = \underline{\underline{\sqrt{\frac{1}{2}}}}$

$g(x) = \frac{x-2}{2x+1/x}$ $\qquad \lim_{x\to\infty} \frac{x-2}{2x+1/x} = \lim \frac{x}{x} \cdot \frac{1-\frac{2}{x}}{2+1/x^2} =$

$f(y) = \sqrt{y}$ $\qquad \overset{\text{VOAL}}{=} \frac{\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{2}{x}}{\lim_{x\to\infty} 2 + \lim_{x\to\infty} 1/x^2}$

$= \frac{1}{2}$

$\lim_{y\to 1/2} \sqrt{y} = \sqrt{1/2}$

P1 $\sqrt{y}$ spoj. v $1/2$

(1) (8) $\lim_{x \to 5} \left| -e^{-1/x} \right| = \frac{1}{\sqrt[5]{e}}$

(b) $f(y) = |y|$

$g(x) = -e^{-1/x}$

(a) $h(x) = -1/x$

$p(y) = -e^{y}$

$\lim_{x \to 5} -e^{-1/x} = -\frac{1}{\sqrt[5]{e}}$

$\lim_{x \to 5} -1/x = -1/5$

$\lim_{y \to -1/5} -e^{y} = -e^{-1/5}$

$= -\frac{1}{\sqrt[5]{e}}$

$\lim_{y \to -\frac{1}{\sqrt[5]{e}}} |y| = \frac{1}{\sqrt[5]{e}}$

P1 $|y|$ spoj na $\mathbb{R}$

P1 $-e^{y}$ spoj v $-1/5$

(1) (a) $\lim_{x \to -1-} \ln \frac{x-1}{x+1} = \infty$

$g(x) = \frac{x-1}{x+1}$

$\lim_{x \to -1-} \frac{x-1}{x+1} = \infty$

$f(y) = \ln y$

$\lim_{y \to \infty} \ln y = \infty$

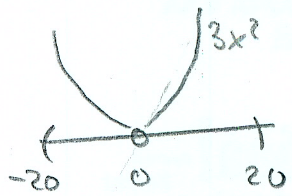
P2 $g(x) \neq \infty$ $\forall x \in \mathbb{R}$ (vyhýbá se své limitě)

(2) (1) $\lim\limits_{x\to 0} \dfrac{\sin 3x^2}{x^2} \overset{\text{VOAL}}{=} \lim\limits_{x\to 0} 3 \cdot \lim \dfrac{\sin 3x^2}{3x^2} = \underline{\underline{3\cdot 1}}$

$g(x) = 3x^2$ $\qquad \lim\limits_{x\to 0} 3x^2 = 0$

$f(y) = \dfrac{\sin y}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\sin y}{y} = 1$

(P1) $3x^2 \neq$ pro $\forall x \in P^{20}(0)$



(2) $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} \sqrt{x}}{\sqrt{2x}} \overset{\text{VOAL}}{=} \lim\limits_{x\to 0} \dfrac{1}{\sqrt{2}} \cdot \lim\limits_{x\to 0} \dfrac{1}{\cos \sqrt{x}} \cdot \lim\limits_{x\to 0} \dfrac{\sin \sqrt{x}}{\sqrt{x}} = \underline{\underline{\dfrac{1}{\sqrt{2}}}}$

- $g(x) = \sqrt{x}$ $\qquad \lim\limits_{x\to 0} \sqrt{x} = 0$ $\qquad$ P2 $\sqrt{x} \neq 0$ pro $x \in P^{10}(0)$

  $f(y) = \dfrac{\sin y}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\sin y}{y} = 1$

- $g(x) = \sqrt{x}$ $\qquad \lim\limits_{x\to 0} \sqrt{x} = 0$ $\qquad$ P1 $\dfrac{1}{\cos y}$ spoj v 0

  $f(y) = \dfrac{1}{\cos y}$ $\qquad \lim\limits_{y\to 0} \dfrac{1}{\cos y} = 1$

(3) $\lim\limits_{x\to 0} \ln\left(\dfrac{x}{\sin x}\right) = 1$

$g(x) = \dfrac{x}{\sin x}$ $\qquad \lim\limits_{x\to 0} \dfrac{x}{\sin x} = \lim \dfrac{1}{\frac{\sin x}{x}} = 1$

$f(y) = \ln y$ $\qquad \lim\limits_{y\to 1} \ln y = 1$

(P1) $\ln y$ spoj v 1

(2) (4) $\lim\limits_{x\to 0} \dfrac{x^4}{1-\cos 4x^2} = \lim\limits_{x\to 0} \dfrac{1}{16} \cdot \lim \dfrac{16x^4}{1-\cos 4x^2} = \dfrac{2}{16} = \underline{\underline{\dfrac{1}{8}}}$

$g(x) = 4x^2$ $\qquad \lim\limits_{x\to 0} 4x^2 = 0$

$f(y) = \dfrac{y^2}{1-\cos y}$ $\qquad \lim\limits_{y\to 0} \dfrac{y^2}{1-\cos y} = \lim\limits_{y\to 0} \dfrac{1}{\frac{1-\cos y}{y^2}} = 2$

(P2) $4x^2 \neq 0$ na $P^{\delta_0}(0)$

(5) $\lim\limits_{x\to 0} x \cdot \operatorname{cotg} 3x = \lim \dfrac{\cos 3x}{\sin 3x} \cdot x$

$= \lim\limits_{x\to 0} \dfrac{\cos 3x}{3} \cdot \lim \dfrac{3x}{\sin 3x} = \dfrac{1}{3} \cdot 1 = \underline{\underline{\dfrac{1}{3}}}$

- $g(x) = 3x$ $\qquad \lim\limits_{x\to 0} 3x = 0$ $\qquad$ P2 $\quad 3x \neq 0$ pro $x \in \mathbb{R}, x \neq 0$

  $f(y) = \dfrac{y}{\sin y}$ $\qquad \lim\limits_{y\to 0} \dfrac{y}{\sin y} = 1$

- $g(x) = 3x$ $\qquad \lim\limits_{x\to 0} 3x = 0$ $\qquad$ P1 $\quad \cos y$ spoj v 0

  $f(y) = \cos y$ $\qquad \lim\limits_{y\to 0} \cos y = 1$

(3)(1) $\lim\limits_{x\to 0} \dfrac{\ln(1+3x)}{x} = \lim\limits_{x\to 0} 3 \cdot \dfrac{\ln(1+3x)}{3x} = 3 \cdot 1$

$g(x) = 3x$ $\qquad \lim\limits_{x\to 0} 3x = 0$

$f(y) = \dfrac{\ln(1+y)}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\ln(1+y)}{y} = 1$

P 2 $\quad 3x \neq 0$ pro $x \neq 0$

(3)(2) $\lim\limits_{x\to\infty} x \cdot \ln\left(1+\dfrac{1}{x}\right) = \lim\limits_{x\to\infty} \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \underline{1}$

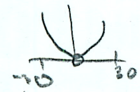
$g(x) = 1/x$ $\qquad \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$

$f(y) = \dfrac{\ln(1+y)}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\ln(1+y)}{y} = 1$

P 2 $\quad \dfrac{1}{x} \neq 0 \quad \forall x \in D_g$

(3)(3) $\lim\limits_{x\to 0} \dfrac{\ln(1+x^2)}{\ln(1-x^2)} = \lim \dfrac{\ln(1+x^2)}{x^2} \cdot \lim \dfrac{-x^2}{\ln(1-x^2)} \cdot (-1)$

- $g(x) = x^2$ $\qquad \lim\limits_{x\to 0} x^2 = 0$ $\qquad$ P 2



$f(y) = \dfrac{\ln(1+y)}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\ln(1+y)}{y} = 1$ $\qquad x^2 \neq 0$ na $P\delta_0$-okolí nuly.

- $g(x) = -x^2$ $\qquad \lim\limits_{x\to 0} -x^2 = 0$ $\qquad$ P2 analogicky

$f(y) = \dfrac{\ln(1+y)}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\ln(1+y)}{y} = 1$

celkem $\lim\limits_{x\to 0} \dfrac{\ln(1+x^2)}{\ln(1-x^2)} = \underline{-1}$

(3)(4)

$$\lim_{x\to\infty} x \ln\left(1-\frac{3}{x}\right) = \lim_{x\to\infty} -3 \cdot \frac{\ln\left(1-\frac{3}{x}\right)}{-3\cdot\frac{1}{x}} = -3\cdot 1 = -3$$

$g(x) = -\frac{3}{x}$ $\qquad \lim_{x\to\infty} \frac{-3}{x} = 0$

$f(y) = \frac{\ln(1+y)}{y}$ $\qquad \lim_{y\to 0} \frac{\ln(1+y)}{y} = 1$

(P2) $\frac{3}{x} \neq 0 \quad \forall x \in D_g$

(3)(5) $\lim_{x\to\infty} x \ln\left(1-\frac{2}{x^2}\right) = \lim_{x\to\infty} x \cdot \frac{-2}{x^2} \cdot \frac{\ln\left(1-\frac{2}{x^2}\right)}{-\frac{2}{x^2}} = 0\cdot 1 = \underline{\underline{0}}$

- $\lim_{x\to\infty} -\frac{2}{x} = 0$

- $g(x) = -\frac{2}{x^2}$ $\qquad \lim_{x\to\infty} -\frac{2}{x^2} = 0$

  $f(y) = \frac{\ln(1+y)}{y}$ $\qquad \lim_{y\to 0} \frac{\ln(1+y)}{y} = 1$

  (P1) $-\frac{2}{x^2} \neq 0 \quad \forall x \in D_g$