

15. cvičení

http://www.karlin.mff.cuni.cz/~kuncova/
kytaristka@gmail.com

Fakta

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Příklady

1. Spočítejte limity zadaných funkcí

(a)

$$\lim_{x \rightarrow 0} \frac{\sin 3x^2}{x^2}$$

(h)

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$$

(b)

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} \sqrt{x}}{\sqrt{2x}}$$

(i)

$$\lim_{x \rightarrow 0} x \cotg 3x$$

($\operatorname{tg} x = \sin x / \cos x$.)

(j)

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{\sin^3 x}$$

(c)

$$\lim_{x \rightarrow 0} \ln \left(\frac{x}{\sin x} \right)$$

(k)

$$\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x}$$

(d)

$$\lim_{x \rightarrow 0} \frac{x^4}{1 - \cos 4x^2}$$

(l)

$$\lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2}$$

(e)

$$\lim_{x \rightarrow 0} x \cotg 3x$$

(m)

$$\lim_{x \rightarrow \frac{\pi}{4}} \operatorname{tg} 2x \operatorname{tg} \left(\frac{\pi}{4} - x \right)$$

($\cot x = \cos x / \sin x$)

(f)

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

(n)

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$$

(g)

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x}$$

(o)

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$$

(p)

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 + \operatorname{tg} x} - \sqrt{1 + \sin x}}{x^3}$$

(q)

$$\lim_{x \rightarrow 0} \frac{x^2}{\sqrt{1 + x \sin x} - \sqrt{\cos x}}$$

(r)

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$$

(s)

$$\lim_{x \rightarrow \pi} \frac{\sin mx}{\sin nx},$$

kde m, n jsou přirozená čísla.