Vaclav

(4)

(1) (a) $\lim_{x\to\infty} \frac{\sin x}{x} = 0$ 2 policajti

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x} \quad \text{na } \mathbb{R}^+$$
$$-\frac{1}{x} \to 0, \quad \frac{1}{x} \to 0$$

(b) $\lim_{x\to\infty} \frac{\ln x}{x^2} = 0$

2 policajti

$$\frac{0}{x^2} \le \frac{\ln x}{x^2} \le \frac{x-1}{x^2}$$
pro $x \in (1, \infty)$

$$\lim_{x\to\infty} 0 = 0 \qquad \lim_{x\to\infty} \frac{x-1}{x^2} = \lim_{x\to\infty} \frac{x^2}{x^2} \frac{\frac{1}{x} - \frac{1}{x^2}}{1} = 0$$

(c) $\lim_{x\to\infty} e^{-x} \cos x = 0$

2 policajti

$$-1 \cdot e^{-x} \le f \le e^{-x} \cdot 1$$
$$-1 \cdot e^{-x} \to 0, \quad e^{-x} \cdot 1 \to 0$$

(d) $\lim_{x\to\infty} \frac{x + \sin x}{x - \sin x} = \lim_{x\to\infty} \frac{x}{x} \cdot \frac{1 + \frac{\sin x}{x}}{1 - \frac{\sin x}{x}} = \underline{1}$ viz 1 c

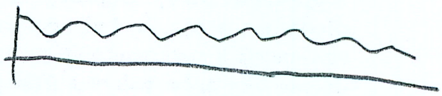
$\frac{\sin x}{x} \to 0$ 2 policajti

(e) $\lim_{x\to 0+} x \cos\left(\frac{x+3}{\sqrt{x}-1}\right) = 0$ 2 policajti

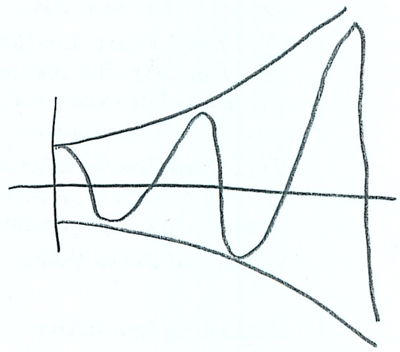
$$-x \le \qquad \le x$$
$$-x \to 0, \quad x \to 0$$

(f) $\lim\limits_{x\to\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}} = \lim\limits_{x\to\infty} \dfrac{e^x}{e^x}\cdot\dfrac{1+e^{-2x}}{1-\underbrace{e^{-2x}}_{\downarrow\, 0}} \overset{\text{vstl}}{=} \underline{\underline{1}}$

(g) $\lim\limits_{x\to\infty} (2+\cos x)$ $\nexists$

"graf:  "

(h) $\lim\limits_{x\to\infty} e^x\cos x$ $\nexists$

"graf:  "

(i) $\lim\limits_{x\to 0} \dfrac{x^2}{e^x} \overset{\text{vstl}}{=} \dfrac{0}{1} = \underline{\underline{0}}$

(j) $\lim\limits_{x\to\infty} \dfrac{x}{\sin x}$ $\nexists$ "- čas od časa delimo 0"

(2)(a) $$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{x}{x}\cdot\frac{\sqrt{1+\frac{1}{x^2}}}{1} \overset{\text{vo+L}}{=} \underline{\underline{1}}$$

$$\sqrt{\lim} = \lim\sqrt{}$$

(b) $$\lim_{x\to\infty} \sqrt{x+2}-\sqrt{x} = \lim_{x\to\infty} \frac{x+2-x}{\sqrt{x+2}+\sqrt{x}} =$$

$$= \lim_{x\to\infty} \frac{2}{\underbrace{\sqrt{x+2}}_{\infty}+\underbrace{\sqrt{x}}_{\infty}} = 0$$

(c) $$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to-\infty} \frac{|x|}{x}\cdot\frac{\sqrt{1+\frac{1}{x^2}}}{1} = \underline{\underline{-1}}$$

(d) $$\lim_{x\to\infty} \sqrt{x+2}+\sqrt{x} = \text{“}\infty+\infty\text{”} = \underline{\underline{\infty}}$$

(e) $$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = \lim_{x\to 0} \frac{x+1-1}{x\,(\sqrt{x+1}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1}$$

$$\overset{\text{vo+L}}{=} \frac{1}{\sqrt{0+1}+1} = \underline{\underline{\frac{1}{2}}}$$

(f) $$\lim_{x\to-2} \frac{\sqrt[3]{x-6}+2}{x^3+8} = \lim_{x\to-2} \frac{x-6+2^3}{(x^3+8)\left(\sqrt[3]{x-6}^{\,2}-2\sqrt[3]{x-6}+2^2\right)}$$

$$= \lim_{x\to-2} \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x^2-x(+2)+4)\left(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4\right)} =$$

$$= \lim_{x\to-2} \frac{1}{\underbrace{x^2}_{4}\underbrace{-2x}_{+4}\underbrace{+4}_{+4}}\cdot\frac{1}{\left(\underbrace{\sqrt[3]{(x-6)}^{2}}_{4}\underbrace{-2\sqrt[3]{x-6}}_{4}\underbrace{+4}_{4}\right)} \overset{\text{vo+L}}{=} \frac{1}{12^2} = \underline{\underline{\frac{1}{144}}}$$

(g) $\lim\limits_{x\to\infty} x\left(\sqrt{x^2+1}-x\right) = \lim\limits_{x\to\infty} \dfrac{x\left(x^2+1-x^2\right)}{\sqrt{x^2+1}+x} =$

$= \lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}+x} = \lim\limits_{x\to\infty} \dfrac{x}{x}\cdot\dfrac{1}{\sqrt{1+\underbrace{\frac{1}{x^2}}_{\downarrow\, 0}}+1} \overset{\text{VoAL}}{=} \underline{\underline{\dfrac{1}{2}}}$

(h) $\lim\limits_{x\to\infty} x^3-x^2+3x-8 = \lim x^3\left(1-\frac{1}{x}+\frac{3}{x^2}-\frac{8}{x^3}\right) = \text{"}\infty\cdot 1\text{"}$

$= \underline{\underline{\infty}}$

(i) $\lim\limits_{x\to\infty} x+\sin x = \infty$

policajti

$x-1 \leq \qquad \leq x+1$