14. cvičení

Hinty

$$A^n - B^n = (A - B)(A^{n-1} + A^{n-2}B + A^{n-3}B^2 + \ldots + A^2B^{n-3} + AB^{n-2} + B^{n-1})$$

Příklady

1. Určete z **definice** následující limity (či jejich neexistenci)

   (a)
   $$\lim_{x \to 0} \frac{1}{x}$$

   (b)
   $$\lim_{x \to \infty} x$$

   (c)
   $$\lim_{x \to \infty} \frac{x}{x+1}$$

2. Spočtěte limity

   (a) $\lim_{x \to \frac{\pi}{4}} \operatorname{tg} x$

   (b) $\lim_{x \to -\infty} (x+3)^2$

   (c) $\lim_{x \to \infty} e^{-x}$

   (d) $\lim_{x \to \infty} \frac{3}{-8-x}$

   (e) $\lim_{x \to 5} \frac{6}{x-5}$

   (f) $\lim_{x \to \infty} \frac{1}{\ln x + 1}$

   (g) $\lim_{x \to 1} \frac{x^2+4x-5}{x-1}$

   (h) $\lim_{x \to 1} \frac{x^2+4x-5}{(x-1)^2}$

   (i) $\lim_{x \to -\infty} \frac{x^3+3x+5+\frac{1}{x}}{8x^3+4x^2-3}$

   (j) $\lim_{x \to \frac{\pi}{2}} \operatorname{tg} x$

   (k) $\lim_{x \to 2} \ln(x-3)$

   (l) $\lim_{x \to 1} \frac{x}{\sqrt{x^2-1}}$

   (m) $\lim_{x \to 0} \frac{x^3-2x+x}{2x^3+x^2-2x}$

3. Spočtěte limity

   (a)
   $$\lim_{x \to \infty} \frac{\sin x}{x}$$

   (b)
   $$\lim_{x \to \infty} e^{-x} \cos x$$

   (c)
   $$\lim_{x \to \infty} \frac{x + \sin x}{x - \sin x}$$

   (d)
   $$\lim_{x \to 0+} x \cos\left(\frac{x+3}{\sqrt{x}-1}\right)$$

   (e)
   $$\lim_{x \to \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

   (f)
   $$\lim_{x \to \infty} (2 + \cos x)$$

   (g)
   $$\lim_{x \to \infty} e^x \cos x$$

   (h)
   $$\lim_{x \to 0} \frac{x^2}{e^x}$$

   (i)
   $$\lim_{x \to \infty} \frac{x}{\sin x}$$

4. Spočtěte limity

(a)
$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x}$$
$$(\lim \sqrt{} = \sqrt{\lim})$$

(b)
$$\lim_{x\to\infty} \sqrt{x+2} - \sqrt{x}$$

(c)
$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x}$$

(d)
$$\lim_{x\to\infty} \sqrt{x+2} + \sqrt{x}$$

(e)
$$\lim_{x\to0} \frac{\sqrt{x+1}-1}{x}$$

(f)
$$\lim_{x\to-2} \frac{\sqrt[3]{x-6}+2}{x^3+8}$$

(g)
$$\lim_{x\to\infty} x(\sqrt{x^2+1} - x)$$

(h)
$$\lim_{x\to\infty} x^3 - x^2 + 3x - 8$$

Bonus

5. Spočtěte limity

(a)
$$\lim_{x\to2} \frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}}$$

(b)
$$\lim_{x\to1} \frac{x^m-1}{x^n-1},$$
kde $m, n \in \mathbb{N}$

(c)
$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$$

(d)
$$\lim_{x\to+\infty} \frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x-1}}$$

(e)
$$\lim_{x\to4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2}$$

(f)
$$\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}},$$

kde $a > 0$

(g)
$$\lim_{x\to-2} \frac{\sqrt[3]{x-6}+2}{x^3+8}$$

(h)
$$\lim_{x\to+\infty} \left(\sqrt{x+\sqrt{x+\sqrt{x}}} - x\right)$$

(i)
$$\lim_{x\to0+} \left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}} - \sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)$$

(j)
$$\lim_{x\to+\infty} x^{1/3}\left[(x+1)^{2/3} - (x-1)^{2/3}\right]$$