(1) $\lim\limits_{x\to\infty} \ln(e^x + x^2 - \ln x) \cdot \arcsin\frac{1}{x+\ln x}$

$$\overset{(**)}{=} \lim \frac{\arcsin\frac{1}{x+\ln x}}{\frac{1}{x+\ln x}} \cdot \frac{1}{x+\ln x} \cdot \overset{(*)}{\ln e^x\left(1+\frac{x^2}{e^x}-\frac{\ln x}{e^x}\right)}$$

$$\overset{\text{VOAL}}{=} 1 \cdot 1 = \underline{\underline{1}}$$

$$\lim \frac{\ln e^x + \ln\left(1+\frac{x^2}{e^x}-\frac{\ln x}{e^x}\right)}{x+\ln x} =$$

$$= \lim \frac{x\left(1+\frac{\ln\left(1+\frac{x^2}{e^x}-\frac{\ln x}{e^x}\right)}{x}\right)}{x\left(1+\frac{\ln x}{x}\right)} =$$

$$\overset{\text{VOAL}}{=} \frac{1+0\infty}{1+0} = 1 \qquad \text{lebo už vieme, že } \lim_{x\to\infty}\frac{\ln x}{x} = 0$$

(*) $\lim\limits_{x\to\infty} \ln\left(1+\frac{x^2}{e^x}-\frac{\ln x}{e^x}\right) = 0$

lebo z VOLSF $f(y) = \ln y$ $\qquad \lim\limits_{y\to 1} f(y) = 0$

(S) ln spoj. v 1 $\qquad g(x) = 1 + \frac{x^2}{e^x} - \frac{\ln x}{e^x}$ $\qquad \lim g(x) \overset{\text{VOAL}}{=} 1+0-0$

známe lim v ∞

(**) VOLSF $f(y) = \frac{\arcsin y}{y}$ $\qquad \lim\limits_{y\to 0} f(y) = 1$

$g(x) = \frac{1}{x+\ln x}$ $\qquad \lim\limits_{x\to\infty} \frac{1}{x+\ln x} \overset{\text{VOAL}}{=} \frac{1}{\infty+\infty} = 0$

(P) $\frac{1}{x+\ln x} \neq 0 \quad \forall x \in (0,\infty)$

(2)

$$\lim_{x\to\frac{\pi}{4}} (\cot g\, x)^{tg\, 2x} = \lim_{x\to\frac{\pi}{4}} e^{tg\, 2x\, (\ln \cot g\, x)} = e^1$$

$f(y) = e^y$ (S) e^y spoj. v 1

$g(x) = tg\, 2x \ \ln \cot x$ $\lim_{y\to 1} e^y = e$

$$\lim_{x\to\frac{\pi}{4}} g(x) = \lim_{x\to\frac{\pi}{4}} \frac{\ln \cot x}{\cot x - 1} \cdot \frac{tg\, 2x}{} \cdot (\cot x - 1)$$

$\overset{VOAL}{=}$ $1 \cdot 1 = 1$

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin 2x}{\cos 2x}\left(\frac{\cos x}{\sin x} - 1\right) = \lim \frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}\left(\frac{\cos x - \sin x}{\sin x}\right)$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{2\cos x}{\cos x + \sin x} = \frac{2\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}} = 1$$

spojitost

$$\lim_{x\to\frac{\pi}{4}} \frac{\ln \cot x}{\cot x - 1}$$

$g(x) = \cot x$ $\lim_{x\to\frac{\pi}{4}} \cot x = 1$

$f(y) = \frac{\ln y}{y-1}$ $\lim_{y\to 1} \frac{\ln y}{y-1} = 1$

(P): $\cot x \neq 1$ na $P^{0,1}\left(\frac{\pi}{4}\right)$

(3)

$$\lim_{x\to 0} \frac{1 - \cos x \cdot \sqrt{\cos 3x}}{\operatorname{arctg}(\arcsin 2x \cdot \sin 3x)} =$$

$$= \lim \frac{\arcsin 2x \cdot \sin 3x}{\operatorname{arctg}(\arcsin 2x \cdot \sin 3x)} \cdot \frac{2x \cdot 3x}{\arcsin 2x \cdot \sin 3x} \cdot$$

$$\cdot \frac{1 - \cos x + \cos x - \cos \sqrt{\cos 3x}}{6x^2}$$

(a) $\lim\limits_{x\to 0} \frac{2x}{\arcsin 2x} = 1$

$g(x) = 2x$ $\qquad \lim\limits_{x\to 0} 2x = 0$

$f(y) = \frac{y}{\arcsin y}$ $\qquad \lim\limits_{y\to 0} \frac{y}{\arcsin y} = 1$

VOLSF, (P) $2x \neq 0$ na $P^{1/2}(0)$

(b) $\lim\limits_{x\to 0} \frac{3x}{\sin 3x} = 1$

VOLSF, analogicky

(c) $\lim\limits_{x\to 0} \frac{\arcsin 2x \cdot \sin 3x}{\operatorname{arctg}(\arcsin 2x \cdot \sin 3x)} = 1$

spojitost arcsin, sin, 2x, 3x

$\Downarrow$

$g(x) = \arcsin 2x \cdot \sin 3x$ $\qquad \lim\limits_{x\to 0} g(x) = 0$

$f(y) = \frac{y}{\arctan y}$ $\qquad \lim\limits_{y\to 0} f(y) = 1$

(P) $g(x) \neq 0$ na $P^{\pi/4}(0)$

(3) (d) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{6x^2} = \dfrac{1}{6}\cdot\dfrac{1}{2}$

(e) $\lim\limits_{x\to 0} \dfrac{\cos x\,(1-\sqrt{\cos 3x})}{6x^2} = \lim \cos x \cdot \dfrac{1-\cos 3x}{9x^2}\cdot\dfrac{9}{6(1+\sqrt{\cos 3x})}$

$\overset{VOAL}{=} \lim\limits_{x\to 0} \cos x \cdot \lim \dfrac{1-\cos 3x}{9x^2}\cdot \lim \dfrac{9}{6}\,\dfrac{1}{1+\sqrt{\cos 3x}} = 1\cdot\dfrac{1}{2}\cdot\dfrac{9}{6}\cdot\dfrac{1}{1+\sqrt{1}}$

$= \dfrac{9}{6\cdot 4} = \dfrac{3}{8}$

(f) $\lim\limits_{x\to 0} \dfrac{1-\cos 3x}{9x^2}$ (P) $3x \neq 0$ na $P^{\delta}(0)$

$g(x) = 3x$ $\lim\limits_{x\to 0} g(x) = 0$

$f(y) = \dfrac{1-\cos y}{y^2}$ $\lim\limits_{y\to 0} f(y) = \dfrac{1}{2}$

(g) $\lim\limits_{x\to 0} \cos x = 1$ (spojitost cos)

(h) $\lim \dfrac{9}{6}\,\dfrac{1}{1+\sqrt{\cos 3x}} = \dfrac{9}{6}\cdot\dfrac{1}{1+1}$ (spojitost $\dfrac{1}{1+\sqrt{\cos 3x}}$ $\rightarrow$ složení spojitých)

(4) $\lim\limits_{x\to 1} \arccos \dfrac{x^{60}-3x+2}{x^{40}-2x+1}$

Idea: když zkusíme dosadit, vyjde $\arccos\frac{0}{0}$. Musí tedy jít vytknout $(x-1)$.

Řešení

$$\lim_{x\to 1} \frac{x^{60}-3x+2}{x^{40}-2x+1} = \lim \frac{x^{60}-x \quad -2x+2}{x^{40}-x \quad -x+1}$$

$$= \lim \frac{x(x^{59}-1)-2(x-1)}{x(x^{39}-1)-(x-1)} = \lim \frac{x(\cancel{x-1})(x^{58}+\dots+1)-2(\cancel{x-1})}{x(\cancel{x-1})(x^{38}+\dots+1)-(\cancel{x-1})\cdot 1}$$

$$\overset{\text{VOAL}}{=} \frac{1\cdot 59-2}{1\cdot 39-1} = \underline{\underline{\frac{57}{38}}}$$

kdybychom teď ale zkusili VOLSF, tak zjistíme, že to nejde

- arccos v $\frac{57}{38}$ ani ve jejich okolí není definován.

$\to$ Limita tedy vůbec není definována a neexistuje.

Konkrétně: $D_{\arccos} = [-1,1]$

jelikož $\lim\limits_{x\to 1} \dfrac{x^{60}-3x+2}{x^{40}-2x+1} = \frac{57}{38} > 1{,}4$

tak ~~xxxx~~ $\exists\,\delta: \forall x \in P^{\delta}(1): \dfrac{x^{60}-3x+2}{x^{40}-2x+1} > 1{,}4$

tedy pro $x \in P^{\delta}(1)$ $\arccos \dfrac{x^{60}-3x+2}{x^{40}-2x+1}$ není definován

a tedy $\lim \nexists$.

(5) $\lim_{x\to\frac{\pi}{4}} \operatorname{tg} 2x \cdot \operatorname{tg}\left(\frac{\pi}{4}-x\right) = \frac{1}{2}$

Idea: uděláme posun do 0, kde známe různé tabulkové limity.

Řešení

$g(x) = \frac{\pi}{4} - x$

Pomocný výpočet: $y = \frac{\pi}{4} - x$

$f(y) = \left(\operatorname{tg} \frac{\pi}{2} - 2y\right) \cdot \operatorname{tg} y$

$\frac{\pi}{4} - y = x$

$\frac{\pi}{2} - 2y = 2x$

$\lim_{x\to\frac{\pi}{4}} g(x) = 0$

(P) $\frac{\pi}{4} - x \neq 0$ na $P^{42}\left(\frac{\pi}{4}\right)$

$$\lim_{y\to 0} f(y) = \lim_{y\to 0} \frac{\operatorname{tg} y}{y} \cdot y \cdot \operatorname{tg}\left(\frac{\pi}{2}-2y\right)$$

$$\overset{\text{VOAL}}{=} \lim \frac{\operatorname{tg} y}{y} \cdot \lim y \cdot \operatorname{tg}\left(\frac{\pi}{2}-2y\right) = 1 \cdot \underline{\underline{\frac{1}{2}}}$$

$$\lim y \cdot \operatorname{tg}\left(\frac{\pi}{2}-2y\right) = \lim_{y\to 0} y \cdot \frac{\overset{\nearrow 1}{\sin\frac{\pi}{2}} \cos(-2y) + \overset{\nearrow 0}{\cos\frac{\pi}{2}} \sin(-2y)}{\underset{\to 0}{\cos\frac{\pi}{2}} \cos(-2y) - \underset{\to 1}{\sin\frac{\pi}{2}} \sin(-2y)}$$

$$= \lim_{y\to 0} y \cdot \frac{\cos(-2y)}{\sin 2y} \overset{\text{VOAL}}{=} \lim \frac{\cos(-2y)}{2} \cdot \lim \frac{2y}{\sin 2y} = \frac{1}{2} \cdot 1$$

spojitost cos v 0

VOLSF $g(y) = 2y$

$f(z) = \frac{z}{\sin z}$

$\lim_{y\to 0} g(y) = 1$

$\lim_{z\to 0} f(z) = 1$

(P) $2y \neq 0$ na $P^{18}(0)$

(6) $$\lim_{x\to 0} \left(2e^{\frac{x}{x+1}}-1\right)^{\frac{x^3+1}{x}} = \lim_{x\to 0} e^{\frac{x^3+1}{x}\ln\left(2e^{\frac{x}{x+1}}-1\right)}$$

$f(y) = e^y$

$$g(x) = \frac{x^3+1}{x}\ln\left(2e^{\frac{x}{x+1}}-1\right)$$

$$\lim_{x\to 0} \frac{x^3+1}{x}\cdot\ln\left(2e^{\frac{x}{x+1}}-1\right) =$$

$$= \lim_{x\to 0} \frac{\ln\left(2e^{\frac{x}{x+1}}-1\right)}{2e^{\frac{x}{x+1}}-1-1}\cdot\frac{2\left(e^{\frac{x}{x+1}}-1\right)}{\frac{x}{x+1}}\cdot\frac{x}{x+1}\cdot\frac{x^3+1}{x}$$

$$\overset{\text{VOAL}}{=} 1\cdot 2\cdot 1\cdot\frac{0+1}{0+1} = \underline{\underline{2}}$$

(a) $f(y) = \dfrac{\ln(y)}{y-1}$ $\qquad \lim_{y\to 1}\dfrac{\ln y}{y-1} = 1$

$g(x) = 2e^{\frac{x}{x+1}}-1$ $\qquad \lim_{x\to 0} g(x) = 2-1 = 1$

spojité fce (složení spoj.) v 0

(P) $2e^{\frac{x}{x+1}}-1 \neq 1$ na $P^{1/2}(0)$

neb $\Leftrightarrow$ $e^{\frac{x}{x+1}} = 1$ $\Leftrightarrow$ $\frac{x}{x+1}\neq 0$ na $P^{1/2}(0)$

$\left(\frac{x}{x+1} = 1+\frac{-1}{x+1}\ \ldots\right)$

(b) $f(y) = \dfrac{e^y-1}{y}$ $\qquad \lim_{y\to 0} f(y) = 1$

$g(x) = \dfrac{x}{x+1}$ $\qquad \lim_{x\to 0}\dfrac{x}{x+1} = 0$

spojitost

(P) $\frac{x}{x+1}\neq 0$ na $P^{1/2}(0)$

(7) $$\lim_{x\to\infty} \underbrace{\frac{\operatorname{arccotg}\frac{x}{\sqrt{1+x^2}}\cdot \operatorname{arccotg}\frac{1}{x}}{\arcsin^2\frac{1}{x}}}_{h(x)} = \infty$$

Rozbor: typ $\frac{\frac{\pi}{4}\cdot\frac{\pi}{2}}{0+}$, $\left(\operatorname{arccotg}\frac{x}{\sqrt{1+x^2}} \to \frac{\pi}{4},\ \operatorname{arccotg}\frac{1}{x}\to\frac{\pi}{2}\right)$

$\arcsin^2\frac{1}{x} \to 0$ typ: $\pm\infty$

neb $\{h(x)\geq 0$, typ: ∞

- $\lim\limits_{x\to\infty} \operatorname{arccotg}\frac{x}{\sqrt{1+x^2}} = \frac{\pi}{4}$

neb $g(x) = \frac{x}{\sqrt{1+x^2}}$ $\quad \lim \frac{x}{x(\sqrt{\frac{1}{x^2}+1})} \overset{(*)}{=} \lim\limits_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = 1$

$f(y) = \operatorname{arccotg} y$ $\quad \lim\limits_{y\to 1} \operatorname{arccotg} y = 1$

(S) spoj. arccot v 1

$*$ $\quad g(x) = 1+\frac{1}{x^2}$ $\quad \lim\limits_{x\to\infty} g(x) = 1+0$

$f(y) = \sqrt{y}$

$\lim\limits_{y\to 1} \sqrt{y} = 1$

(S) spoj. $\sqrt{y}$ v 1

- $\lim \operatorname{arccotg}\frac{1}{x} = \frac{\pi}{2}$

$\lim\limits_{x\to\infty}\frac{1}{x} = 0$

$\lim\limits_{y\to 0} \operatorname{arccot} y = \frac{\pi}{2}$ $\quad$ (S) arccot y spoj v 0

~~Pak~~ - $\lim \arcsin^2\frac{1}{x} = 0$

$\lim\limits_{x\to\infty}\frac{1}{x} = 0$ $\quad$ (S) $\arcsin^2 y$ spoj v 0

$\lim\limits_{y\to 0} \arcsin^2 y = 0$

Závěr: $\lim \quad = 0$ neb $\arcsin^2 y \geq 0$ a z lim výše

(8) $\displaystyle\sum_{n=2}^{\infty} \underbrace{\left(\ln \frac{n-1}{n+1}\right)\left(\sqrt{n+1}-\sqrt{n}\right)^{p}}_{a_n}$ $\qquad p \in \mathbb{R}$

(1)

$a_n = \underbrace{\ln\left(1+\frac{-2}{n+1}\right)}_{\leq 0} \cdot \underbrace{\left(\frac{1}{\sqrt{n+1}+\sqrt{n}}\right)^{p}}_{\geq 0}$ $\to$ budeme zkoumat $\sum -a_n = -\sum a_n$

srovnáme s $b_n = \frac{2}{n+1} \cdot \frac{1}{(\sqrt{n})^{p}}$; $\lim_{n\to\infty} \frac{-a_n}{b_n} =$

$= \lim_{n\to\infty} \frac{-\ln\left(1+\frac{-2}{n+1}\right)}{\frac{2}{n+1}} \cdot \frac{(\sqrt{n})^{p}}{(\sqrt{n+1}+\sqrt{n})^{p}} = \lim \frac{\ln\left(1-\frac{2}{n+1}\right)}{-\frac{2}{n+1}} \cdot \frac{1}{\left(\sqrt{1+\frac{1}{n}}+1\right)^{p}}$

$\overset{\text{VoAL}}{=} 1 \cdot \frac{1}{2^{p}}$

(a) Heine $x_n = \frac{-2}{n+1}$ $\quad x_n \to 0 \quad x_n \neq 0 \quad \forall n \in \mathbb{N}$

$f(x) = \frac{\ln(1+x)}{x}$ $\quad \lim_{x\to 0} f(x) = 1$

(b) Heine $x_n = n$, $x_n \to \infty$, $x_n \neq \infty$ $\forall n \in \mathbb{N}$

$\lim_{x\to\infty} \frac{1}{\left(\sqrt{1+\frac{1}{x}}+1\right)^{p}} \overset{\text{VoAL}}{=} \frac{\lim 1}{\lim(\qquad)^{p}}$

(b.1) $f(y) = y^{p}$ $\qquad \lim_{y\to 2} y^{p} = 2^{p}$ (S) spojitost y^{p} ve 2

$g(x) = \sqrt{1+\frac{1}{x}}+1$ $\qquad \lim_{x\to\infty} g(x) = 2$

(b.2) $f(y) = \sqrt{y}$ $\qquad \lim_{y\to 1} \sqrt{y} = 1$ (S) spoj $\sqrt{y}$ v 1

$g(x) = 1+\frac{1}{x}$

$\lim_{x\to\infty} 1+\frac{1}{x} = 1+0 = 1$

(8)

(2) tedy $\sum -a_n$ K $\Leftrightarrow$ $\sum b_n$

$$\sum b_n \ K \overset{(*)}{\Leftrightarrow} \sum \frac{1}{n \cdot n^{p/2}} = \sum \frac{1}{n^{1+p/2}} \ K$$

$$\Leftrightarrow \quad 1 + p/2 > 1$$

$$p/2 > 1$$

$$p > 2$$

<u>Závěr</u> $\sum a_n$ K pro $p > 2$

a D pro $p \leq 2$

(*) LSK b_n a $\frac{1}{n \cdot n^{p/2}}$

$$\lim_{n \to \infty} \frac{\frac{2}{(n+1)\, n^{p/2}}}{\frac{n}{n^{p/2}}} = \lim \frac{2n}{n+1} = \lim \frac{2}{1 + 1/n} \overset{\text{VOAL}}{=} \underline{2}$$

(a) $\sum_{n=1}^{\infty} \underbrace{\frac{1+\sin\frac{1}{n}-\cos\frac{1}{n}}{1+\sin\frac{2}{n}-\cos\frac{2}{n}} \cdot \frac{1}{n}\sqrt{\operatorname{tg}\frac{1}{n}}}_{a_n \geq 0}$

srovnáme s $b_n = \frac{1}{n}\cdot\sqrt{\frac{1}{n}} = \frac{1}{n^{3/2}}$

$$\lim_{n\to\infty} \frac{1+\sin\frac{1}{n}-\cos\frac{1}{n}}{1+\sin\frac{2}{n}-\cos\frac{2}{n}} \cdot \frac{\sqrt{\operatorname{tg}\frac{1}{n}}}{\sqrt{\frac{1}{n}}}$$

Heine $x_n = \frac{1}{n}$, $x_n \to 0$, $x_n \neq 0$ $\forall n \in \mathbb{N}$

$$\lim_{x\to 0} \sqrt{\frac{\operatorname{tg} x}{x}} \cdot \frac{1+\sin x-\cos x}{1+\sin 2x-\cos 2x} \overset{\text{VOAL}}{=} 1 \cdot \frac{1}{2}$$

$$\lim_{x\to 0} \frac{x\left(\frac{1-\cos x}{x}+\frac{\sin x}{x}\right)}{x\left(\frac{1-\cos 2x}{x}+\frac{\sin 2x}{x}\right)} = \lim_{x\to 0} \frac{x\frac{1-\cos x}{x^2}+\frac{\sin x}{x}}{(4x)\frac{(1-\cos 2x)}{4x^2}+2\cdot\frac{\sin 2x}{2x}}$$

$$\overset{\text{VOAL}}{=} \frac{0\cdot\frac{1}{2}+1}{4\cdot 0\cdot\frac{1}{2}+2\cdot 1} = \frac{1}{2}$$

Závěr: neb $\sum b_n$ k $\Rightarrow$ $\underline{\underline{\sum a_n \text{ k}}}$

(*) $f(y) = \sqrt{y}$ $\quad \lim_{y\to 1} f(y) = 1$ $\quad$ (S) spoj $\sqrt{y}$ v 1

$g(x) = \frac{\operatorname{tg} x}{x}$ $\quad \lim_{x\to 0} g(x) = 1$

(**) $f(y) = \frac{1-\cos y}{y^2}$ $\quad \lim_{y\to 0} f(y) = \frac{1}{2}$

$g(x) = 2x$ $\quad \lim_{x\to 0} 2x = 0$ $\quad$ (P) $2x \neq 0$ na $P^{42}(0)$

(***) $f(y) = \frac{\sin y}{y}$ $\quad \lim_{y\to 0} f(y) = 1$

$g(x) = 2x$ $\quad \lim_{x\to 0} 2x = 0$

(10) $\displaystyle\sum_{n=1}^{\infty} \underbrace{\frac{n^{n+\frac{1}{n}}}{\left(n+\frac{1}{n}\right)^n}}_{a_n}$

$$a_n = \frac{n^n \cdot n^{\frac{1}{n}}}{n^n\left(1+\frac{1}{n^2}\right)^n} = \frac{n^{\frac{1}{n}}}{\left[\left(1+\frac{1}{n^2}\right)^{n^2}\right]^{\frac{1}{n}}}$$

$$\lim_{n\to\infty} a_n = \lim \frac{\sqrt[n]{n} \to 1}{\underbrace{\sqrt[n]{\underbrace{\left(1+\frac{1}{n^2}\right)^{n^2}}_{\downarrow\, e}}}_{\to 1}} \neq 0$$

dohromady

nesplňuje NP, $\sum$ diverguje

Podrobněji

$$\lim \frac{\sqrt[n]{n}}{\sqrt[n]{\left(1+\frac{1}{n^2}\right)^{n^2}}} \overset{\text{věta}}{=} \frac{\lim \sqrt[n]{n}}{\lim \sqrt[n]{\left(1+\frac{1}{n^2}\right)^{n^2}}} = \frac{1}{1} = 1$$

$\lim \sqrt[n]{n} = 1$ (známá lim)

$\lim \left(1+\frac{1}{n^2}\right)^{n^2} = e$ neboť z Heineho

$x_n := n^2$ $\quad x_n \to \infty$ $\quad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$ (známá lim)

tedy $\lim \left(1+\frac{1}{n^2}\right)^{n^2} = e$

tedy $\exists n_0 : \forall n \geq n_0$ $\quad 2 \leq \left(1+\frac{1}{n^2}\right)^{n^2} \leq 3$

ze z policajtů $\quad \underset{\downarrow\, 1}{\sqrt[n]{2}} \leq \sqrt[n]{\left(1+\frac{1}{n^2}\right)^{n^2}} \leq \underset{\downarrow\, 1}{\sqrt[n]{3}}$

tedy i $\sqrt[n]{\left(1+\frac{1}{n^2}\right)^{n^2}} \to 1$

(10) ∃ naš

Nejde

$$\sum_{n=1}^{\infty} \frac{n^{n+\frac{1}{n}}}{(n+\frac{1}{n})^n}$$

Cauchyho krit.

$$\lim_{n\to\infty} \frac{\sqrt[n]{n^n \cdot n^{1/n}}}{n+\frac{1}{n}} = \lim_{n\to\infty} \frac{n \cdot n^{\frac{1}{n^2}}}{n+\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{n^2}}}{\underset{\downarrow 1}{1} + \underset{\downarrow 0}{\frac{1}{n^2}}} \overset{\text{VOAL}}{=} \frac{1}{1+0} = 1$$

$$\lim_{n\to\infty} n^{\frac{1}{n^2}} = \lim \sqrt[n]{\underbrace{\sqrt[n]{n}}_{\to 1}}$$

jelikož $\sqrt[n]{n} \to 1$, tak $\exists n_0 : \forall n \geq n_0$ platí

$$\frac{1}{2} \leq \sqrt[n]{n} \leq 2$$

pak $\underset{\to 1}{\sqrt[n]{\frac{1}{2}}} \leq \sqrt[n]{\sqrt[n]{n}} \leq \underset{\to 1}{\sqrt[n]{2}}$

$$n^{\frac{1}{n^2}} : \quad x^{\frac{1}{x^2}} = e^{\frac{1}{x^2}\ln x} = 1$$