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MFF

(1) (a) 3
(b) 5
(c) 2
(d) 4
(e) 6
(f) 6
(g) 1
(h) 1

(2) (a) $\frac{\pi}{6}$
(b) $-\frac{\pi}{3}$
(c) 0
(d) $\frac{\pi}{2}$
(d) $\frac{\pi}{3}$
(e) $-\frac{\pi}{4}$
(g) $-\frac{\pi}{3}$
(h) $\frac{\pi}{3}$

(3) $\sin x = \frac{1}{2}$

$x_1 = \frac{\pi}{6}$

$x_2 = \frac{5}{6}\pi$

celem $x \in \left\{\frac{\pi}{6} + 2k\pi;\ k \in \mathbb{Z}\right\} \cup \left\{\frac{5}{6}\pi + 2k\pi,\ k \in \mathbb{Z}\right\}$

(4)

(a) $2\arccos\left(\frac{x}{2}-1\right)$

$H_f = [0, 2\pi]$

$D_f: -1 \leq \frac{x}{2} - 1 \leq 1$

$-1 \leq \frac{x}{2} - 1$ $\qquad$ $\frac{x}{2} - 1 \leq 1$ $\qquad$ $D_f = [0, 4]$

$0 \leq x$ $\qquad$ $x \leq 4$

(b) $5 + 2\arccos\sqrt{1-x^2}$

$H_f = [5, 5+2\pi]$

$D_f: \underbrace{-1 \leq}_{\text{triviálně}} \sqrt{1-x^2} \leq 1$

$\sqrt{1-x^2} \leq 1$ $\qquad$ $1-x^2 \geq 0$

$1-x^2 \leq 1$ $\qquad$ $1 \geq x^2$

$0 \leq x^2$ ✓ $\qquad$ $x \in [-1, 1]$

(c) $y = \operatorname{arctg}\frac{1}{x}$

$D_f = \mathbb{R} \setminus \{0\}$

$H_f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \setminus \{0\}$

(d) $\ln(\operatorname{arctg}(1-2x))$

$\operatorname{arctg}(1-2x) > 0$ $\qquad$ $D_f = \left(-\infty, \frac{1}{2}\right)$

tedy $1-2x > 0$

$\frac{1}{2} > x$

$H_f: \left(-\infty, \frac{1}{2}\right) \xrightarrow{1-2x} (0, \infty) \xrightarrow{\operatorname{arctg}} \left(0, \frac{\pi}{2}\right) \xrightarrow{\ln} \left(-\infty, \ln\frac{\pi}{2}\right)$

tedy $\left(-\infty, \ln\frac{\pi}{2}\right)$

–) arcsin $(\ln x)$

D_f : · $x > 0$

· $-1 \le \ln x \le 1$ $\qquad$ $\underline{x \in [\frac{1}{e}, e]}$

$\frac{1}{e} \le x \le e$

H_f : $\underline{[-\frac{\pi}{2}, \frac{\pi}{2}]}$

(f) $\operatorname{arctg} \dfrac{1}{\sqrt{1-\sqrt{x}}}$

D_f : · $x \ge 0$

· $1 - \sqrt{x} > 0$ $\qquad$ $\underline{x \in [0, 1)}$

$1 > \sqrt{x}$

$1 > x$

H_f :

$[0, 1) \xrightarrow{\sqrt{x}} [0, 1) \xrightarrow{1-y} (0, 1] \xrightarrow{\sqrt{z}} (0, 1]$

$\xrightarrow{\frac{1}{u}} [1, \infty) \xrightarrow{\operatorname{arctg}} [\frac{\pi}{4}, \frac{\pi}{2})$

$H_f = \underline{[\frac{\pi}{4}, \frac{\pi}{2})}$

(5)

(a) $y = 2\arccos\left(\frac{x}{2} - 1\right)$

definiční obory jsou H_f z příkladu výše

$$\frac{y}{2} = \arccos\left(\frac{x}{2} - 1\right)$$

$$\cos\frac{y}{2} = \frac{x}{2} - 1$$

$$\underline{\underline{2\left(\cos\frac{y}{2} + 1\right) = x}}$$

(b) $5 + 2\arccos\sqrt{1 - x^2}$

funkce není prostá (může za to "x^2") → nemá inverzi

(c) $y = \operatorname{arctg}\frac{1}{x}$

$$\operatorname{tg} y = \frac{1}{x}$$

$$\underline{\underline{x = \operatorname{cotg} y}}$$

(d) $y = \ln\left(\operatorname{arctg}(1 - 2x)\right)$

$$e^y = \operatorname{arctg}(1 - 2x)$$

$$\operatorname{tg}(e^y) = 1 - 2x$$

$$2x = 1 - \operatorname{tg}(e^y)$$

$$x = \underline{\tfrac{1}{2}\left(1 - \operatorname{tg}(e^y)\right)}$$

(e) $y = \arcsin(\ln x)$

$$\sin y = \ln x$$

$$\underline{\underline{e^{\sin y} = x}}$$

(f) $y = \operatorname{arctg}\frac{1}{\sqrt{1 - \sqrt{x}}}$

$$\operatorname{tg} y = \frac{1}{\sqrt{1 - \sqrt{x}}}$$

$$\sqrt{1 - \sqrt{x}} = \operatorname{cotg} y$$

$$1 - \sqrt{x} = \operatorname{cotg}^2 y$$

$$1 - \operatorname{cotg}^2 y = \sqrt{x}$$

$$\underline{\underline{(1 - \operatorname{cotg}^2 y)^2 = x}}$$

(6) (a) $\operatorname{arctg} x = \operatorname{arccotg} \frac{1}{x}$ $\quad x > 0$

nechť $\operatorname{arctg} x = y$, $\quad y \in (0, \frac{\pi}{2})$

$\iff \quad x = \operatorname{tg} y$

pak ale $\quad \frac{1}{x} = \operatorname{cotg} y$

a $\quad \operatorname{arccotg} \frac{1}{x} = y \quad (= \operatorname{arctg} x)$

(b) $\arcsin x = \arccos \sqrt{1-x^2}$ $\quad x \in [0, 1]$

nechť

$\arcsin x = y$

$\iff \quad x = \sin y \qquad y \in [0, \frac{\pi}{2}]$

$x^2 = 1 - \cos^2 y$

$\cos^2 y = 1 - x^2$

$\cos y = + \sqrt{1-x^2}$

$y = \arccos \sqrt{1-x^2}$