9 Konvergence Newtonova integrálu

Úloha 1. Pro které hodnoty parametrů $a, b \in \mathbb{R}$ integrál konverguje? Je-li to možné, vyčíslete příslušný integrál, v opačném případě využijte srovnávacího kritéria.

a) $\displaystyle\int_0^1 x^a\,dx$, b) $\displaystyle\int_1^\infty x^a\,dx$, c) $\displaystyle\int_0^\infty x^a + x^b\,dx$,

d) $\displaystyle\int_0^{1/e} \frac{|\ln x|^a}{x}\,dx$, e) $\displaystyle\int_1^e \frac{\ln^a x}{x}\,dx$, f) $\displaystyle\int_e^\infty \frac{\ln^a x}{x}\,dx$,

g) $\displaystyle\int_0^{1/e} x^a|\ln x|^b\,dx$, h) $\displaystyle\int_1^\infty x^a e^{bx}\,dx$, i) $\displaystyle\int_0^{\pi/2} \operatorname{tg}^a x\,dx$.

Úloha 2. Rozhodněte o konvergenci následujících integrálů ($p, q \in \mathbb{R}, n \in \mathbb{N}$).

a) $\displaystyle\int_0^\infty x^{-3/4}e^{-\sqrt{x}}\,dx$, b) $\displaystyle\int_0^1 \frac{\operatorname{tg} x}{\sqrt{x^3}}\,dx$, c) $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{1-x^4}}\,dx$,

d) $\displaystyle\int_0^1 x^{\ln x}\,dx$, e) $\displaystyle\int_0^1 x^{-\ln x}\,dx$, f) $\displaystyle\int_0^\infty \frac{\sin\frac{1}{x}\operatorname{arctg} x}{x}\,dx$,

g) $\displaystyle\int_0^\pi \ln\sin x\,dx$, h) $\displaystyle\int_0^\pi \frac{\sin(\sec x)}{\sqrt{x}}\,dx$, i) $\displaystyle\int_0^{1/2} \frac{\arcsin(x^2+x^3)}{x\ln^2(1+x)}\,dx$,

j) $\displaystyle\int_1^2 \frac{\operatorname{arctg}(x-1)}{(x-\sqrt{x})^p}\,dx$, k) $\displaystyle\int_0^\pi \frac{\ln\sin x}{x\sqrt{\sin x}}\,dx$, l) $\displaystyle\int_0^\pi \frac{\sin^p x}{e^{-x^2/2}-\cos x}\,dx$,

m) $\displaystyle\int_0^\infty \frac{x-\sin x}{x^p}\,dx$ n) $\displaystyle\int_0^\infty \frac{\operatorname{arctg} px}{x^n}\,dx$, o) $\displaystyle\int_0^{\pi/2} \sin^p x\cos^q x\,dx$.

Úloha 3. Vyšetřete neabsolutní i absolutní konvergenci následujících integrálů ($\alpha \in \mathbb{R}$).

a) $\displaystyle\int_0^\infty \frac{\sin x}{x^\alpha}\,dx$, b) $\displaystyle\int_0^\infty \frac{\cos x}{x^\alpha}\,dx$, c) $\displaystyle\int_0^\infty \frac{x\sin x}{1+x}\,dx$,

d) $\displaystyle\int_0^\infty \frac{\sin(x+x^2)}{1+x}\,dx$, e) $\displaystyle\int_0^\infty \sin x^\beta\,dx,\ \beta>1$, f) $\displaystyle\int_0^\infty \sin(\operatorname{arccotg} x)\sin x\,dx$,

g) $\displaystyle\int_0^1 \frac{1}{x\sqrt{x}}\cos\frac{\sqrt{x}-1}{\sqrt{x}}\,dx$, h) $\displaystyle\int_0^{1/2} \frac{\cos^3\ln x}{x\ln x}\,dx$, i) $\displaystyle\int_0^\infty x^\alpha\ln(1+x)\cos x\,dx$,

j) $\displaystyle\int_0^{\pi/4} \sin\left(\frac{1}{\sin x}\right)\frac{dx}{\sin^\alpha x}$, k) $\displaystyle\int_0^{\pi/4} \operatorname{tg}^\alpha x\cos(\operatorname{cotg} x)\,dx$, l) $\displaystyle\int_{-1}^1 \sin\frac{1+x}{1-x}\,\frac{dx}{(1-x^2)^\alpha}$,

m) $\displaystyle\int_0^{\pi/2} \operatorname{tg}^\alpha x\sin\frac{1}{x}\,dx$, n) $\displaystyle\int_0^1 x^\alpha\operatorname{arctg} x\cos\frac{1}{x}\,dx$, o) $\displaystyle\int_0^1 \frac{x^\alpha}{e^x-1}\sin\frac{1}{x}\,dx$.