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③ $F(x,y) = e^y f \cdot \left(y e^{\frac{x}{y}}\right) = e^y \cdot f(u)$

$u(x,y) = y \cdot e^{\frac{x}{y}}$

$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial u} (y e^{\cdot})$

$$\frac{\partial F}{\partial x} = e^y \cdot \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} = e^y \cdot f'(u) \cdot y \cdot e^{\frac{x}{y}} \cdot \frac{1}{y} = e^y \cdot f'(u) \cdot e^{\frac{x}{y}} \checkmark$$

$$\frac{\partial F}{\partial y} = e^y \cdot f(u) + e^y \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} = e^y f(u) + e^y f'(u)\left(e^{\frac{x}{y}} + y \cdot e^{\frac{x}{y}} \cdot \frac{x}{y^2} \cdot (-1)\right) =$$

$$= e^y f(u) + e^y f'(u) e^{\frac{x}{y}} - e^y f'(u) e^{\frac{x}{y}} \cdot \frac{x}{y} \checkmark$$

$$\frac{\partial F}{\partial y} + \frac{\partial F}{\partial x} \cdot \left(\frac{x}{y} - 1\right) = e^y f(u) + e^y f'(u) e^{\frac{x}{y}} - e^y f'(u) e^{\frac{x}{y}} \cdot \frac{x}{y} + e^y f'(u) e^{\frac{x}{y}} \cdot \frac{x}{y} - e^y f'(u)$$

$\boxed{10}$

$$= e^y f(u) = F$$

① $$\int_0^1 \ln\left(\sqrt{1+x^2} - x\right) dx = \int_1^{\sqrt{2}-1} \ln y \cdot \frac{-y^2-1}{2y^2} dy \checkmark = -\int_1^{\sqrt{2}-1} \ln y \, \frac{y^2+1}{2y^2} dy =$$

$$= -\frac{1}{2} \int_1^{\sqrt{2}-1} \ln y \, \frac{y^2+1}{y^2} dy$$

$$= -\frac{1}{2} \int_1^{\sqrt{2}-1} \ln y + \frac{\ln y}{y^2} dy$$

$$= -\frac{1}{2} \left(\int_1^{\sqrt{2}-1} \ln y \, dy + \int_1^{\sqrt{2}-1} \frac{\ln y}{y^2} dy \right)$$

$u' = 1 \quad u = y$

$v = \ln y \quad v' = \frac{1}{y}$

$u' = \frac{1}{y^2} \quad u = -\frac{1}{y}$

$v = \ln y \quad v' = \frac{1}{y}$

$y = \sqrt{1+x^2} - x$

$y^2 + 2xy + x^2 = 1 + x^2$

$2xy = 1 - y^2$

$x = \frac{1-y^2}{2y} \checkmark$

$0: y = \sqrt{1+0} - 0 = 1$

$1: y = \sqrt{1+1} - 1 = \sqrt{2} - 1$

$$dx = \frac{-2y(2y) - 2(1-y^2)}{4y^2} dy = \frac{-2y^2 - 1 + y^2}{\cancel{4} 2y^2} dy = \frac{-y^2-1}{2y^2} dy$$

$$= -\frac{1}{2}\left(\left[y \ln y \right]_1^{\sqrt{2}-1} - \int_1^{\sqrt{2}-1} 1 \, dy + \left[-\frac{1}{y} \ln y \right]_1^{\sqrt{2}-1} - \int_1^{\sqrt{2}-1} -\frac{1}{y^2} dy \right) =$$

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$$= -\frac{1}{2}\left(\left[y \ln y \right]_1^{\sqrt{2}-1} - \left[y \right]_1^{\sqrt{2}-1} + \left[-\frac{1}{y} \ln y \right]_1^{\sqrt{2}-1} - \left[\frac{1}{y} \right]_1^{\sqrt{2}-1} \right) = -$$

$= \ln(\sqrt{2}-1)^{\frac{1}{\sqrt{2}} - \frac{3}{2}}$

$\cdot \frac{1}{2\sqrt{2}-2}$

$= \ln(\sqrt{2}-1)^{-2+\frac{1}{\sqrt{2}}}$

$$= -\frac{1}{2}\left((\sqrt{2}-1)\ln(\sqrt{2}-1) - 0 - (\sqrt{2}-1) + 1 + \left(-\frac{1}{\sqrt{2}-1} \ln(\sqrt{2}-1) \right) - 0 - \frac{1}{\sqrt{2}-1} + 1 = -\frac{1}{2}\left(\ln \frac{1}{(\sqrt{2}-1)^2} - \sqrt{2} + 3 - \frac{1}{\sqrt{2}-\ldots} \right.\right.$$

(2) $\displaystyle\int_0^1 \frac{x-\ln(1+x)}{x^{5/2}(1-x)^{1/3}}\,dx = \int_0^{1/2} \frac{x-\ln(1+x)}{x^{5/2}(1-x)^{1/3}}\,dx + \int_{1/2}^1 \frac{x-\ln(1+x)}{x^{5/2}(1-x)^{1/3}}$

$T^{\ln(1+x)}_{n,0} = 0 + \frac{1}{1!}(x)^1 - \frac{1}{2!}x^2 + \frac{1}{3!}\cdot 2\cdot x$

$(\ln(1+x))' = \frac{1}{1+x}$

$\left(\frac{1}{1+x}\right)' = \frac{1}{(1+x)^2}\cdot -1\cdot 1$

$\left(\frac{1}{(1+x)^2}\right)' = \frac{1}{1+x^3}\cdot +2$

$= 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$

v nule

$\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} + \mathcal{O}(x^4)$

integrál se chová jako

$$\int_0^{0,5} \frac{x-\ln(1+x)}{x^{5/2}}\,dx \approx \int_0^{1/2} \frac{x - x + \frac{x^2}{2} + \mathcal{O}(x^3)}{x^{5/2}}\,dx =$$

$$\approx \int_0^{1/2} \frac{\frac{x^2}{2}}{x^{5/2}}\,dx = \frac{1}{2}\int_0^{1/2} x^{-1/2}\,dx$$ → Absolutně konverguje a je kladná

$$\Rightarrow \text{LSk} \quad \lim_{x\to 0} \frac{\frac{x-\ln(1+x)}{x^{5/2}(1-x)^{1/3}}}{\frac{1}{\sqrt{x}}} = \lim_{x\to 0} \frac{x-\ln(1+x)}{x^2}\cdot\frac{1}{(1-x)^{1/3}} \overset{\text{VOAL}}{=}$$

$$\lim_{x\to 0} \frac{x-\ln(x+1)}{x^2}\cdot \underbrace{\lim_{x\to 0}\frac{1}{(1-x)^{1/3}}}_{1} = \lim_{x\to 0}\frac{x-\ln(1+x)}{x^2} \underset{\frac{0}{0}}{\overset{\text{L'H}}{=}} \lim_{x\to 0} \frac{1-\frac{1}{1+x}}{2x} =$$

$$= \lim_{x\to 0}\frac{\frac{1+x-1}{1+x}}{2x} = \lim_{x\to 0}\frac{1}{2(x+1)} = \frac{1}{2}$$ ⇒ LSk vyšlo a v nule integrál absolutně konverguje ✓

v 1

nahoře 1-ln2 to nám nevadí

dole $(1-x)^{1/3}$

vidím, že $\lim_{x\to 1} \frac{\frac{x-\ln(1+x)}{x^{5/2}(1-x)^{1/3}}}{\frac{1-\ln 2}{(1-x)^{1/3}}} = 1$ a funkce je kladná

LSk s $\displaystyle\int_{1/2}^1 \frac{1-\ln 2}{(1-x)^{1/3}}\,dx$

substituce

$y = 1-x$

$dy = -1dx$

$x = 1/2 : y = 1/2$

$x = 1 : y = 0$

$$= -\int_{1/2}^0 \frac{1-\ln 2}{y^{1/3}}\,dy = \int_0^{1/2}\frac{1-\ln 2}{y^{1/3}}\,dy = (1-\ln 2)\int_0^{1/2}\frac{1}{y^{1/3}}\,dy$$

a to konverguje absolutně

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⇒ celý integrál konverguje absolutně