$$F(\alpha) = \int_0^\infty \frac{1-e^{-\alpha x}}{xe^x}$$

Derivace I

(1) Rozbor

$$f = \frac{1-e^{-\alpha x}}{xe^x}$$

$[a,b) = [0,\infty)$

$J = ?$

(A) pro jaká $\alpha$ $\int_0^\infty \frac{1-e^{-\alpha x}}{xe^x}\,dx$ K ?

(a) probl. body : $0, \infty$

$0$: u 0 lze spojitě dodefinovat ✓

(neb $\alpha = 0 \to f \equiv 0$

$\alpha \neq 0$ : $\lim_{x\to 0} \frac{1-e^{-\alpha x}}{xe^x} = \frac{1-e^{-\alpha x}}{-\alpha x} \cdot \frac{-\alpha}{e^x} = -\alpha$

$\infty$: pro $\alpha = 0$ $f \equiv 0$

$\alpha \neq 0$ LSK, $[a,b) = [10,\infty)$, $g(x) = \frac{e^{-\alpha x}}{xe^x}$

$\alpha < 0$ $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{1-e^{-\alpha x}}{e^{-\alpha x}} = -1$

pak $\int_{10}^\infty f(x)\,dx$ K $\iff$ $\int_{10}^\infty g(x)\,dx$ K

ale $\int g(x)$ $\iff$ $-\alpha - 1 < 0$ tedy $\boxed{-1<\alpha}$

$\alpha > 0$

$g(x) = \frac{1}{xe^x}$

$\lim_{x\to\infty} \frac{f}{g} = \lim 1 - e^{-\alpha x} = 1$

$\int g$ K tedy $\int f$ K

tedy $\underline{J = (-1,\infty)}$

(2) Podm. • $f$ je spoj na $(0,\infty) \times (-1,\infty)$

ale my chceme spoj na $[0,\infty)$ → nevadí, frci lze spojitě dodefinovat

• $\int f$ K taktéž $J$ (právě jsme ukázali)

• $\frac{\partial f}{\partial y} = \frac{+xe^{-\alpha x}}{xe^x} = +\frac{e^{-\alpha x}}{e^x}$ spojitá ✓

$$e^{(-\alpha-1)x} \leq \underbrace{e^{-(p+1)x}}_{\text{majoranta } g(x)} \quad \text{pro} \quad \begin{array}{l} p\in(-1,\infty) \\ \alpha\in[p,\infty) \\ x\in(0,\infty) \end{array}$$

(opravdu je konvergentní)

(3) Výpočet $F'(\alpha) = \frac{d}{dy}\int_0^\infty \frac{1-e^{-\alpha x}}{xe^x}\,dx = \int_0^\infty \frac{\partial}{\partial y}\frac{1-e^{-\alpha x}}{xe^x}\,dx$

$$= \int_0^\infty e^{(-\alpha-1)x}\,dx = \frac{1}{\alpha+1}$$

pak $F(\alpha) = \int \frac{1}{\alpha+1}$

$$F(\alpha) = \ln(\alpha+1) + k$$

kolik je $k$?

dosadíme $\alpha = 0$, pak

$$F(0) = \int_0^\infty 0\,dx = 0$$

$$0 = \ln(1) + k$$

$$k = 0$$

tedy

$$\underline{\underline{F(\alpha) = \ln(\alpha+1)}} \qquad \alpha\in(-1,\infty)$$