Limita posloupnosti - komplexní úloha XII

Určete limitu posloupnosti

$$\lim_{n\to\infty} \frac{2^n\sqrt{n^2+n}-n\sqrt{4^n+1}}{\sqrt[n]{2^{n^2}+1}}.$$

Řešení

$$\lim_{n\to\infty} \frac{2^n\sqrt{n^2+n}-n\sqrt{4^n+1}}{\sqrt[n]{2^{n^2}+1}} = \lim_{n\to\infty} \frac{2^n\cdot\sqrt{n^2\cdot\left(1+\frac{1}{n}\right)}-n\cdot\sqrt{2^{2n}\cdot\left(1+\frac{1}{4^n}\right)}}{\sqrt[n]{2^{n^2}\cdot\left(1+\frac{1}{2^{n^2}}\right)}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{2^n}\cdot n\cdot\left(\sqrt{1+\frac{1}{n}}-\sqrt{1+\frac{1}{4^n}}\right)}{\cancel{2^n}\cdot\sqrt[n]{1+\frac{1}{2^{n^2}}}} \cdot \frac{\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)}{\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)} = \lim_{n\to\infty} \frac{n\cdot\left(\cancel{1}+\frac{1}{n}-\cancel{1}-\frac{1}{4^n}\right)}{\sqrt[n]{1+\frac{1}{2^{n^2}}}\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)} =$$

$$= \lim_{n\to\infty} \frac{1-\frac{n}{4^n}}{\sqrt[n]{1+\frac{1}{2^{n^2}}}\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)}$$

VOAL

2 policajti
věta o odmocnině limity

$$\underline{\underline{\frac{1}{2}}}$$

- $-\frac{n}{4^n} \to 0$

- Věta o dvou policajtech

$$\cancel{\sqrt[n]{1+\frac{1}{2^{n^2}}}}$$

$$\sqrt[n]{1} \leq \sqrt[n]{1+\frac{1}{2^{n^2}}} \leq \sqrt[n]{2} \quad \to \sqrt[n]{1+\frac{1}{2^{n^2}}} \to 1$$

- Věta o odmocnině limity

$$\lim_{n\to\infty}\sqrt{1+\frac{1}{n}} = \sqrt{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)} = 1$$

$$\lim_{n\to\infty}\sqrt{1+\frac{1}{4^n}} = \sqrt{\lim_{n\to\infty}\left(1+\frac{1}{4^n}\right)} = 1$$