Limita posloupnosti - komplexní úloha XII

Určete limitu posloupnosti

$$\lim_{n\to\infty} \frac{2^n \sqrt{n^2+n} - n\sqrt{4^n+1}}{\sqrt[n]{2^{n^2}+1}}.$$

Řešení

$$\lim_{n\to\infty} \frac{2^n \sqrt{n^2+n} - n\sqrt{4^n+1}}{\sqrt[n]{2^{n^2}+1}} = \lim_{n\to\infty} \frac{2^n \cdot \sqrt{n^2\cdot\left(1+\frac{1}{n}\right)} - n\cdot\sqrt{2^{2n}\cdot\left(1+\frac{1}{4^n}\right)}}{\sqrt[n]{2^{n^2}\cdot\left(1+\frac{1}{2^{n^2}}\right)}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{2^n}\cdot n\cdot\left(\sqrt{1+\frac{1}{n}} - \sqrt{1+\frac{1}{4^n}}\right)\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)}{\cancel{2^n}\cdot\sqrt[n]{1+\frac{1}{2^{n^2}}}\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)} = \lim_{n\to\infty} \frac{n\cdot\left(1+\frac{1}{n}-1-\frac{1}{4^n}\right)}{\sqrt[n]{1+\frac{1}{2^{n^2}}}\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)} =$$

$$= \lim_{n\to\infty} \frac{1-\frac{n}{4^n}}{\sqrt[n]{1+\frac{1}{2^{n^2}}}\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{1}{4^n}}\right)} \overset{\text{VOAL}}{=} \underline{\underline{\frac{1}{2}}}$$

2 policajti
věta o odmocnině limity

- $-\frac{n}{4^n} \to 0$

- Věta o dvou policajtech

$\cancel{\sqrt[n]{1+\frac{1}{2^{n^2}}}}$

$$\sqrt[n]{1} \leq \sqrt[n]{1+\frac{1}{2^{n^2}}} \leq \sqrt[n]{2} \to \sqrt[n]{1+\frac{1}{2^{n^2}}} \to 1$$

- Věta o odmocnině limity:

$$\lim_{n\to\infty}\sqrt{1+\frac{1}{n}} = \sqrt{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)} = 1$$

$$\lim_{n\to\infty}\sqrt{1+\frac{1}{4^n}} = \sqrt{\lim_{n\to\infty}\left(1+\frac{1}{4^n}\right)} = 1$$