XIII

$$\lim \sqrt[n]{n^n+n^{n+1}+\dots+n^{2n}}\cdot\frac{1}{n^2} \overset{\text{VOAL}}{=} \lim\sqrt[n]{n^2+n^{n+1}+\dots+n^{2n}}\cdot\underbrace{\lim\frac{1}{n^2}}_{0} =$$

$$\lim\sqrt[n]{n^n+n^{n+1}+\dots+n^{2n}}\cdot 0 =$$

$n^n\cdot n^n$

$n^{2n}\cdot x - n^n$

$x = \frac{n^n}{n^{2n}} = \frac{n^n}{}$

$$\frac{(n+1)^{n+1}+(n+1)^{n+2}+\dots+(n+1)^{2n+2}}{n^n+n^{n+1}+\dots+n^{2n}} = \frac{(n+1)^{2n+1}}{(n)^{2n}}\cdot\frac{1+\frac{1}{(n+1)}+\dots}{1}$$

$x = n^n$

$n^{2n}\cdot n^n = n^n$

$n^{2n+1}\cdot n^n$

$2n+1 = n$

$1 = n$

$n^{-n} = \frac{1}{n^n}$

$$\sqrt[n]{n^{2n}\left(\frac{1}{n^n}+\frac{1}{n}+\dots+1\right)} = n^2\cdot\sqrt[n]{\underset{\downarrow 0}{\frac{1}{n^n}}+\underset{\downarrow 0}{\frac{1}{n}}+\dots+1}$$

$$\frac{n^2}{n^2}+\sqrt[n]{1}$$

$$\lim\sqrt[n]{n^n+n^{n+1}+\dots+n^{2n}}\cdot\frac{1}{n^2} = \lim\sqrt[n]{n^{2n}\left(\underset{\downarrow 0}{\frac{1}{n^n}}+\underset{\downarrow 0}{\frac{1}{n}}+\underset{\downarrow 0}{\dots}+\underset{\downarrow 1}{1}\right)}\cdot\frac{1}{n^2} =$$

$$\cancel{= \lim n^2\cdot\sqrt[n]{1}\cdot\frac{1}{n^2} = \lim\frac{n^2}{n^2}\cdot\sqrt[n]{1} \overset{\text{VOAL}}{=} \lim 1\cdot\underbrace{\lim\sqrt[n]{1}}_{1} =}$$

$$= \boxed{1}$$

$$\lim\sqrt[n]{\left(\frac{1}{n^n}+\frac{1}{n}+\dots+1\right)}\cdot n^2\cdot\frac{1}{n^2} \overset{\text{VOAL}}{=} \lim\sqrt[n]{\left(\frac{1}{n^n}+\frac{1}{n}+\dots+1\right)}\cdot\underbrace{\lim\frac{n^2}{n^2}}_{=1} = 1\cdot 1$$

$$\Rightarrow \text{VOZP} \Rightarrow \left(\frac{1}{n^n}+\frac{1}{n}+\dots+1\right) \le n \Rightarrow \lim\sqrt[n]{n} = 1$$

pretože každý člen okrem posledného je menší ako 1 a ich počet je rovnaký ako počet n-členov

$$1 \le \left(\frac{1}{n^n}+\frac{1}{n}+\dots+1\right) \le n \Rightarrow \text{každý} \le 1$$

$$\lim\sqrt[n]{1} \le \sqrt[n]{\left(\frac{1}{n^n}+\frac{1}{n}+\dots+1\right)} \le \sqrt[n]{n}$$

$$1 \le \lim\sqrt[n]{\frac{1}{n^n}+\frac{1}{n}+\dots 1} \le \lim\sqrt[n]{n} = 1$$