Limita posloupnosti - komplexní úloha V

Určete limitu posloupnosti

$$\lim_{n\to\infty} \sqrt[n]{n^2 + n^3 + n^4 + 2^n + 3^n + 4^n}.$$

Řešení

$$L = \lim \sqrt[n]{n^2+n^3+n^4+2^n+3^n+4^n} = \lim \sqrt[n]{4^n\left(\underbrace{\frac{n^2}{4^n}}_{\to 0} + \underbrace{\frac{n^3}{4^n}}_{\to 0} + \underbrace{\frac{n^4}{4^n}}_{\to 0} + \underbrace{\left(\frac{1}{2}\right)^n}_{\to 0} + \underbrace{\left(\frac{3}{4}\right)^n}_{\to 0} + 1\right)} =$$

$$-1 < \frac{1}{2} < 1 \qquad -1 < \frac{3}{4} < 1$$

$$\cancel{= \lim 4 \cdot \sqrt[n]{0+0+0+0+0+1} = \lim 4 \cdot \sqrt[n]{1} = 4}$$

$$= \lim 4 \cdot \sqrt[n]{\frac{n^2}{4^n} + \frac{n^3}{4^n} + \frac{n^4}{4^n} + \left(\frac{1}{2}\right)^n + \left(\frac{3}{4}\right)^n + 1}$$

věta o dvou policajtech

$$\overset{\text{VO4L}}{\geq} 4 \cdot \lim \sqrt[n]{1} = 4 \cdot 1 = 4$$

$$\overset{\text{VO4L}}{\leq} 4 \cdot \lim \sqrt[n]{42} = 4 \cdot 1 = 4$$

$$4 \leq L \leq 4$$

$$\boxed{L = 4}$$