(VIII)

$$\lim \left[\sqrt[4]{n^4+4n^3} - n\right]$$

$[\ ]$ – celá část; a_n

$$= \lim \frac{\sqrt[4]{n^4+4n^3}^{\,2} - n^2}{\sqrt[4]{n^4+4n^3}+n} = \frac{n^4+4n^3-n^4}{\left(\sqrt[4]{n^4+4n^3}+n\right)\left(\sqrt[4]{(n^4+4n^3)^2}+n^2\right)}$$

$n^2 = \sqrt[4]{n^8}$

$$\sqrt[4]{(n^4+4n^3)^2} = \sqrt[4]{n^8+8n^7+16n^6}$$

$$= \lim \frac{4n^3}{\sqrt[4]{n^{12}+12n^{11}+48n^{10}+64n^9}+n^3+\sqrt[4]{n^{12}+8n^{11}+16n^{10}}+\sqrt[4]{n^{12}+4n^{11}}}$$

$$= \lim \frac{4n^3}{n^3\left(\sqrt[4]{\left(1+\frac{4}{n}\right)^3}+\sqrt[4]{\left(1+\frac{4}{2}\right)^2}+\sqrt[4]{1+\frac{4}{n}}+1\right)} \quad r_n$$

$$r_n > 4 \Rightarrow \frac{4}{r_n} < 1 \Rightarrow \left[\frac{4}{r_n}\right] = 0 \Rightarrow \lim a_n = 0$$

$\forall n$ $\quad \frac{4}{r_n} > 0$